OSAKA ECONOMIC PAPERS

VOL. III MARCH 1955 NUMBER 3

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POWER AND ECONOMY
—SOME COUNTER-CRITICISMS ON POWER THEORY IN ECONOMICS—

YASUMA TAKATA

I

Social power in general may be divided into (a) economic power, (b) non-economic power; power meaning the ability of any one person to compel obedience to his will in any social activity. In the case of economic power, obedience is maintained by delivering some economic goods in exchange so that compelled obedience is indirect. In the case of non-economic power, obedience is maintained directly without delivering any material means; for example, by physical force, political coercion, legal privilege, customary subordination or influences through prestige and authority. When we say " power versus economy " or " power and economy ", we use the terms in the sense of non-economic power.

The relation between power and economy has been treated and analyzed repeatedly by many authors, and there prevails a general and authoritative opinion in the field of economic science: that economy is autonomous. Naturally, the system of free economy is presupposed.

A definite organization of social powers, political, customary and intermental, must be given as a condition to any social economy. Once given, it develops according to its own logic, that is, along economic laws which may be derived from one general principle, that of maximum satisfaction. Consequently, there is no room left to treat the operations of powers in the realm of economic theory, the function of which is nothing other than ideologically to catch the autonomus movement of economic reality. Whether conscious of it or not, the general current of such ideas may be termed the Monroe Doctrine of economic theory prevailing among scholars of the present day.

Against the orthodox line of thinking, we find another group of heretic opinions which may be summarized under the name of power theories in economics (Machttheorien). The authors belonging to this school, though small in number, insist that power plays such an important role in economy that economy cannot be autonomous in the sense that it has no relations with power, and that it is always inevitably determined by power. As a representative doctrine in this direction, we find the social theory of distribution of Tugan-Baranowsky. Sometimes the economic theory of Franz Oppenheimer is said to belong to the school, but we cannot accept this view. Though he stressed the importance of class relations in admitting that all prices in a capitalistic society were determined as the prices of class-monopoly, he did not dare to deny the autonomy of the formation of monopoly prices in general by which wage and interest as class-monopoly prices are necessarily formed. Accordingly power as the class relation is dissolved into monopoly as an economic category. Consequently, we cannot count his economic theory as a kind of power theory.

Of course it is just to say that Eugen Dühring is a representative figure of the school after Hobbes and Macchiavelli, but his theories are too vague and rough being constructed before the appearance of utility theory which forbids us making them objects of scientific analysis. Thus it must be permitted to admit the so-called social theory of Tugan-Baranowsky as most representative among power theories of this century. But his reasoning seems to us to be lukewarm; he accepts the orthodox utility theory concerning the prices of products with no reserve or hesitation. His criticisms and resistances concentrate upon the field of distribution so that the distribution of income, and therefore prices of factors, must be explained by power relation. Fairly to speak, he stands yet on the stage before the general acceptance of the interdependence theory of Lausanne. We must admit that the determination of the prices of productive factors, as well as of the prices of products must be explained taking into consideration the factor of power, since the general equilibrium theory has been once accepted.

Recently the theoretical importance of power, not merely bargaining power but power in general, has become more and more acknowledged, especially in the sphere of labor economics. But the problem and its investigation has always been restricted to wage and labor and not enlarged to the social economy in general which implies the narrowness of the scientific visual circle. If wages are determined by power relations to some degree, then the whole situation of a social economy must be also determined by that relation to the same degree in general since the principle of propagation will prevail. More clearly to say, all prices of products must adapt themselves to the level of wage as the fundamental element of costs. Then, too, the time has come to re-examine the probable validity of power theory in general and if necessary to reformulate present price theory which has seemed to be a kind of axioms since the days of Leon Walras.

Now it will be the just and the shortest way of the re-examination of power theory to take up and analyze Tugan's trial and thereafter to review the famous refutation of Schumpeter directed to ward power theory in general and especially to ward Tugan's standpoint.

II. *Tugan's ways of reasoning*

We meet with a kind of dichotomy in the theoretical system of Tugan. The whole field of economics is divided into (a) value and price theory and (b) distribution theory. The phenomena of value and price are formed between buyers and sellers, both belonging to the same class. Value and price do not contain the element of class inequality or strong and weak relations but the very facts of distribution exists between capitalists and laborers. Under the interference of power relation wages take the form of prices. The theory of values and prices has developed through some stages and attained the marginal utility doctrine which is valid for the whole field of value. But the categories of distribution such as wage and profit cannot be explained either by utility theory or by the labor value theory of Marx, since these facts are formed between inequals. The orthodox theory has derived the unity of economic theories, by transforming distribution categories into price categories so that wage and profit are not grasped as such but are treated as prices. And the criticisms above are also valid literally for the Marxian theory of value.

"Die Verteilungsphänomene sind, nach ihrer inneren Natur, keine Wertphänomene, obschon sie in Wertform erscheinen. Dieser Satz sollte die Grundlage einer wirklich wissenschaftlichen sozialen Theorie der Verteilung werden".[1] According to his view, sellers and buyers of labor belong to different social classes respectively. Therefore, the theory of wages cannot be validly constructed if class relations are neglected.

"Aber bei den Mieten eines Arbeiters müssen die Teilnehmer dieses Tauschaktes nach der Natur dieses Aktes selbst zu verschiedenen sozialen Klassen gehören. Soziale Macht und Abhängigkeitsverhältnisse bei diesem Tauschakte zu ignorieren, heisst den Kern des betreffenden Problems ausschalten".[2]

As the consequence of such a situation, the general level of wages is explained neither by the law of demand and supply, nor by the cost of production theory.[3] The process of distribution of social products among different social classes is not a process of simple exchange but the complex result of class struggles, struggles between highly unequal participants. This inequality is the necessary foundation of wage and profit because poverty only makes laborers sell their labor.[4]

Then he obtained the following conclusion. The average level of wages is determined as resultant effect of two factors: (a) the productivity of social labor that is national product, and (b) the social power of the laboring class which determines "labors' relative share" in social products.[5]

To say frankly, Tugans' power theory must be appreciated as a pioneers' work which detected and attacked severely the defects of traditional wage theory, not as a finished and elaborated system of thoughts. First of all, we profess that he rejected unduely the demand and supply analysis of wage though we are not the last to acknowledge the speciality of labor supply. Secondly, he remained in the field of macro-analysis of distribution and did not take the trouble of micro-analysis to explain the level of wage in the labor market. Thirdly, the dichotomy, that is the division of economic theory into two parts, that of prices and that of distribution, may not be justified since the prices of products must be equally influenced by power relation because the former are determined by the costs of production, the fundamental part of which consists of wages. Two spheres, the prices of products and the prices of productive services, may not be clearly cut, in as much as they are intimately interdependent. Fourthly, his last conclusion that the general level of wages is always determined by total products on one side and by relative power situations of laborer classes on the other is nothing new and original; perhaps it is the cultural inheritance of the German historical school.[6]

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[1] Tugan-Baranowsky, Soziale Theorie der Verteilung, 1913, S. 24.

[2] ibiden, S. 33.

[3] ibid., S. 39.

[4] ibid., S. 43.

[5] ibid., S. 43.

Notwithstanding all these objections, we profess that the Tugans' power theory of economics contains some kernel of truth which we cannot reject nor forget; wages are determined ultimately by power relation so that any and all prices are reflexes of social powers. Of course we do not deny the traditional theory and consequently also the utility analysis of demand and supply, so that we have to synthetize this analysis with the power theory of wages at the first step.

But before we go on along in this direction, we must accomplish some preliminary work to analyze the criticisms thrown upon Tugans' power theory, by such an eminent theorist as Joseph Schumpeter. If we cannot succeed in the rejection of them, we will not be qualified to insist upon our thesis.

III. *Schumpeters' Criticisms*

In entering into the criticisms of Schumpeter we are reminded of the epoch making rejection which Böhm-Bawerk cast upon the power theory of Stolzmaun which seems rather rough and prescientific. Since we must understand the Schumpeterian criticisms along the line of current Australian thoughts, we shall not neglect to follow Böhms' sharp insights in relation to this point.

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[6] Gustav Schmoller, Allgemeine Volkswirtschaftslehre, 2, Teil, 1906, passim.

In his famous article " Macht oder ökonomisches Gesetz " his reasoning runs as the following. Once the capitalistic system is presupposed, there prevails the autonomous economic laws so that the requirement of maximum principle is satisfied. If the action of power makes the actual economy deviate from the laws, there will necessarily occur a turning-back movement toward the line of the laws, until the prevalence is restored. To take an example, suppose the wage rises in consequence of the trade union pressure. Substitution of capital for labor will happen so that the demand for labor decreases, and consequently unemployment becomes inevitable. When the association of union is loose, individual unemployed members will propose to cut down wages. When it is solid, wage bill (the sum of wages received by members) will decrease so that the welfare of union must necessarily decrease, owing to the decreasing return of roundaboutness of production. Counter movement will happen to increase employment with the consequence that all are employed at previous wage rate. The rising of wage by trade union pressure cannot continue long. Thus the principle of the inefficiency of power in economy must be acknowledged. This Böhm-Bawerkian demonstration rejected prefectly any form of power theory of the earlier stage of this century. This line of thought is acceptable, it seems to us, so far as we think in the narrow frame of free economy which differs always from the actual economy. Of course the principle of inefficiency of power war accepted by Schumpeter, but he did not remain in the same frame and considered the problem from higher and wider viewpoint. So the views of Schumpeter on this point is not clear and simple as in the case of Böhm-Bawerk. Further to say, it is very complex that it seems ambiguous in some cases.

It is needless to say that Schumpeter, according to the Austrian tradition of maximization principle, did not acknowledge the efficiency of power in determining wage-level and consequently in changing the supply side of all commodities. On this point, his view did not differ from that of Böhm Bawerk above mentioned. But he did not insist that power can make no impact upon the course of social economy in any sense, but what he intended to say is as follows.

Powers are given data which make up the social frame in which the economic reality develops itself according to its autonomous laws. Then what is the real meaning of given data? Concerning this point, it is too tiresome work to cite and analyze various passages from his articles. We can satisfy by giving the outline of thought.

(1) Non-economic power forms the socio-economic organization in which the economic activities develop themselves. Thus the organization determines the individual possessions or properties and respective demands of social strata, receiving as historical inheritance the technique and capital of production.

These data being given, prices and quantities of products are necessarily determined by pure-economic process, that is to say, all economic quantities are determined according to maximization principle. There reign the autonomous laws of economy. Thus the primary actions of non-economic powers on economy are threefold. Power determines ($\dot{1}$) organizations (Produktionsverhältnisse in the Marxian sense) as the frame of economic activity in general; ($\dot{2}$) together with initial quantities pertaining to competing individuals; ($\dot{3}$) further with the free route of economic activities, that is, metaphorically expressed, the channel of actions in the pure-economic direction. The functions of power ($\dot{1}$) ($\dot{2}$) ($\dot{3}$) make up the frame for pure economic actions, by cooperating together. This is the meaning in which Schumpeter says that powers are given conditions for exchange or free economy.

(2) Non-economic powers interfere with the course of free autonomous development of economy, through the control of the state which is realized to the highest degree in the various forms of collectivism and directed economy, and to the least degree in the fixed prices, control of demand and supply concerning some commodities which are not rare even in the so-called free system, especially in monetary policies. These

are the examples of the deviation from the autonomous course through the pressure of legal powers.

(3) Non-economic powers give influences upon the actual course of economy through social pressures, sanctioned by tradition or founded by social current, rooted in ideology or guided by real forces, organized or inorganized. In any case, there is conceivable a hypothetical free price-zone which would be determined by the hypothetical passive atitudes of both partners, buyers and sellers, which means that A does not actively require what to gain but receives only what B delivers in exchange what A offers and vice versa. In comparing the actual exchange ratio with the hypothetical free price, we can estimate the deviation by pressure. This deviation is not so considerable and important in the case of transaction of products, but very conspicuous in the demand and supply of labors. But as a theoretically important case, it is realized regulary in the payment for work and service. Labor is a special goods which cannot be separated from the personality of supplier so that the price or remuneration may differ according to the condition of social pressure irrespectively of its productivity.

Now I feel the need to make clear the position of "bargaining power". In what relation with non-economic power does it stand? The concept of bargaining is not incompatible with free competitions, or with autonomous course of economy. In free socity, there are many cases where the conditions of perfect competition are not satisfied, such as bilateral monopoly, oligopoly, monopolistic competition. In these cases, one equilibrium position is impossible to realize. The margins are given at any point between which the price may be settled according to the relation of bargaining powers of both parties. In this case, buyer and seller stand on the equal standpoint as contractors and the role of bargaining power is to move the price to the most respectively favourable position. The strength of the power in case is determined by relative needs of transaction and individual tactics, one rational and calculable, the other irrational and non-predictable. The bargaining power in such case is the proper or purely economic one, and beyond this we can mention another kind of bargaining power which will be named bargaining power in the wider sense or disguised bargaining power, in which the good are inseparable from the person; and non-economic power intrudes into transaction so that the exchange ratio is modified to the benifit of the stronger. Though buyers are many and sellers also many so that the condition seems to be fulfiled to make market perfectly competitive, the result of transaction may be indeterminate, and the chain of bargaining will determine the price actually. In such case, bargaining power is the unification of non-economic power plunged into the realm of economic transaction, and proper and pure economic bargaining power which is the function of relative needs and tactics. Disguised bargaining power is, in its essential content, nothing other than the transplanted non-economic power in economic transaction though the transaction takes the form of bilateral monopoly, imperfect competitions or oligopoly in appearance. But in this case the kind of bargaining power is not restricted in its sphere of operation by the extent of rational calculation of gain and loss from the point of which it has been studied since the days of Cournot. Especially in the field of labor dispute concerning wage raising the irrational element of industrial warfare awakes to operate.

To sum up, (1) non-economic power operates as the precondition which constructs the system and organizations to make possible the so-called autonomous economy. In the extent of this frame pure economic action plays their role perfectly free. (2) means that the state regulates, hinders, or promotes the flow of economic activities as non-economic powers. (3) shows that non-economic powers, apart from state intervention, are always penetrating into every day transactions so that all the concrete economic activities are coloured with non-economic power operating outside of state, that is between individuals or between groups, of which we are always unconscious but by which influenced generally.

After long excursus, we have arrived the stage where we may analyze the Schumpeterian criticisms of power theories.

According to his view, the determination of economic quantities are influenced by many causes or groups of causes (Ursachengruppe).

Among these causes there is the economic principle as one which determines the said quantities directly and other causes which also determine them in their respective ways.

Social power is also one of them, and climate, for example, is another. To say that social power determines economy is equally right and equally wrong as to say climate determines economy. About the actions of a distinct group of causes a theory may be formulated which is methodologically autonomous, and in which the other groups of causes are treated merely as given data or preconditions.[7] And the same may be performed about other group of causes. Now take up only two groups of causes, the economic causes which the orthodox economic theory treats and concerns and the social power or the (sozial-rechtliche) social-legal causes. It is not permissible to think that economic principle and social power determines economic quantities side by side and consequently not permissible to explain the economic reality as the result of two coordinated causes, power and economic laws. The methodological autonomy is important; among different series of problems corresponding group of causes, one series must be selected and treated, explanation being performed by one group of causes while other groups are treated as given data. The relation between power and economic principle may not be conceived as side by side action (nebeneinanderwirken) along the same line but as the after-one-another action (nacheinanderwirken) of different chains of social causations, correspondings respective group of causes. The parts, that is, different chains interact upon one another, but methodologically the actions of one cause-group selected are viewed as autonomous. The actions of others are then treated and considered in order to make clear the economic theory, but not to perform sociological synopsis. Take up an analogy. In playing card, the results depend on two factors or causes. On one side, the distribution which is fixed by fate; on the other side, the skill of players. The final result is not determined only by one factor; the explanation by the factor is necessarily partial and insufficient. The very relation between distribution and skill seems to be the same as the relation between power and economic laws.[8]

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[7] op. cit., pp. 16–17.

Let us recapitulate the above mentioned reasoning. What does determine the economic quantities? It is not merely the economic laws (consequently the economic principle) which plays most important role. If A is taken up and its effects are considered, B must be set aside as given data and the methodologically autonomous theories are formed through such consideration. If B is taken up instead of A, another autonomous theory may be formed likewise. If the determination of economic quantities by economic principle is questioned, then social power will be put aside as given data and its action do not come into the front.

Such Schumpeterian treatment of social power is based upon the two grounds. (1) When the actions of one group of causes upon the economy are questioned, other groups must be conceived merely as given data. (2) Power does not belong to a group of causes, pertaining to economic laws.

IV. *Methodological autonomy*

Böhm-Bawerkian criticisms of power theory intends to shut it out from economic theory owing to its inefficiency on the course of economy. Since the action of power, for example, of trade union to raise up wage ultimately ends in vain so that the economic quantities are determined only by the economic laws. In this case the autonomy means autonomy in the material sense. In contrast to this, Schumpeterian criticisms intends equally to shut it out from economic theory by the methodological reason. He does not deny the efficiency of power on the real course of economy, and admits that its action is penetrating into economic life rather generally. But when once a group of causes underlying economic laws is taken into consideration, the other causes are put aside as given data, so that the theory tracing the effects of the former is independent and self-sufficient, that is, autonomous. The autonomy of orthodox economic laws is merely the matter of method and it must be perfectly synonymous with the method of isolations of any theoretical science. Thus the economic laws are must be autonomous in the same sense and in the same degree as the explanation of economy by power or by climate. But we can not help finding some confusion. There are some great differences between the power theory of economy and climate theory of economy. Of course both may be autonomous in the methodological sense. But I must ask what is the characteristic of social sciences? Surely it is the explanation by motive, therefore by understanding. The pure economics (the orthodox economics) stand on the basis of the principle of maximum satisfaction, the free economic system and non-power-seeking subjects being presupposed. Power theory of economy intends to explain by the same motive and presupposing the free economic system and power seeking subjects. Both are the same kind of knowledge, the explanation of economy by motive and understanding. The distinction between the two lies in the difference of ideal-types (models). The Schumpeterian concept of climate theory of economy will be at most the description of characteristics in the economy of some region. In such a situation, it seems peculiar to put power and climate on the same level.

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[8] op. cit., p. 26.

To sum up, what is given by Schumpeter's methodological analysis of the power theory of economy? (1) The theory is a simply one of many possible autonomous economic theories which stand on a respective group of causes. And among many possible economic sciences including climate economics, the traditional or orthodox economics is conceived to consist of "economic laws" which is put in contrast to power theory. By what reason? In this point Schumpeter's ways of thinking is not clear. (2) He does not deny that power exercises influences on economy in manifold ways, so that the pure economy which is the object of study in orthodox economics is a mere marginal case, or a fictitious model. Then the economy coloured by power must also be the object of study as another marginal case, that is, a free system of economy with power seeking subjects. To accomplish such a study means tracing the action of a group of causes, which are common in large extent to those of orthodox economics, but a little different from them. For example, in the orthodox theory, causes are a, b, c; in the power theory a, b, c and x. Possibility of the power theory which is very near with orthodox theories may be proved; what discriminates a power theory from the orthodox theory is in mere difference of grouping of causes which will be determined by the efficiency in explanation.

V. *Conclusions*

After long digressions we arrive at the following conclusion.

Any economic theory presupposes a definite type of social economy constructed by fictions often named ideal type, pure type, pure model, which is at small or large distance from economic reality. Because of its fictional character, the attained economic laws may have general validity.

Type A. Free economic system in which all economic subjects act perfectly following the principle of exchange, that is, one receives what he demands in exchange what he supplies, non economic power being inactive. Therein all subjects do not seek power, only maximizing satisfaction. I name such social economy "utility economy". (1) Exchange rate or term of trade is determined as an equilibrium point as in competition; (2) or, as a zone of possibilities as in oligopoly etc. In the latter case, the pure bargaining power will act seeking maximum gain, the result will depend on relative needs and tactics. The economic cosmos of Léon Walras and of nearly all orthodox economists belong this type.

Type B. A legally and formally free system is presupposed but one in which subjects seek satisfaction by using actions of wider range, that is acting in power seeking attitude. He does not wait what others deliver in exchange but aims at maximum of higher grade using non-economic power. The characteristic of Type B is most significant in the recent trend of labor unions. Since the days of "General Theory" Type B in its mature form is being realized; this type in its earlier form prevailed until last century, to the benefit of the capitalists. When we take up the power-seeking tendency into consideration, the economic theory enters into a new structure treating as causes the maximum satisfaction through non-economic power under definite legal system. And this is the road to successive approximation in the sense classified by Vilfredo Pareto. Without depending on this complication, the economic theory in its traditional form cannot explain the era of labor supremacy of the present as well as the capital supremacy in earlier capitalism. Economic theory as the first approximation in the orthodox form seems to me a land of dreams far from the economic reality full of peaceful conflicts and warfare of economic and non-economic powers.

POSITIVE AND NEGATIVE RELATIONS AND STABILITY CONDITIONS

MASAZO SONO

PREFACE

Discussion on stability conditions of market equilibrium has been one of the problems in economic theory which has interested me since I first read Leon Walras' *Elements d'Economie Politique Pure.* Professor J. R. Hicks discussed the matter in the frame-work of the general equilibrium theory in his *Value and Capital* of 1939. Since I found his discussions not fully satisfactory, I wrote a paper entitled "Stability Conditions of the Market Equilibrium" dated November 1943, which was published in February of 1944 in *Keizai Ronso* by Kyoto University (in Japanese). Unfortunately, wartime conditions did not permit the paper to be circulated abroad. Later I found that Professor P. A. Samuelson's conclusions as published in his "The Stability of Equilibrium: Statics and Dynamics", (*Econometrica*, for April 1941), and *Foundations of Economic Analysis*, 1947, closely parallelled my own. At this time I am convinced that there is yet another method of approach to the discussion of stability of market equilibrium in dynamic theory than that used by Professor Samuelson. I therefore present the following notes for the scrutiny of professional economists. The stability conditions of the changing price system will be handled in an additional paper to be published at a later date.

I. POSITIVE AND NEGATIVE RELATIONS

1. *Two Kinds of Stability.*

In the discussion of stability conditions in economic theory two kinds of stability must be distinguished. The first is the stability of demand and supply which is usually dealt with in the equilibrium theory. The second is the stability of the deviation of price system from equilibrium. This is the problem of stability to be solved in dynamic theory. Leon Walras discussed the stability conditions of the second kind, offering an analogy of the movement of a pendulum. I shall discuss mathematically these two problems.

2. *Positive and Negative Variations.*

Let $y_1, y_2, \dots, y_n$ be functions of independent variables $x_1, x_2, \dots, x_n$ which are the continuous and differentiable:

$$(1) \qquad y_i = f_i(x_1, x_2, \dots, x_n) \qquad (i = 1, 2, \dots, n)$$

Let also the variations of the functions for the infinitesimal changes of the variables:

$$(2) \qquad dx_1, dx_2, \dots, dx_n,$$

be expressed by

$$(3) \qquad \Delta y_1, \Delta y_2, \dots, \Delta y_n.$$

In order to distinguish between the positive and negative changes of the functions we shall take a space of n dimentions in which the line element ds is defined by

$$(4) \qquad ds^2 = dx_1^2 + dx_2^2 + \dots + dx_n^2.$$

Then the angle θ between two vectors $(dx_1, dx_2, \dots, dx_n)$ and $(\Delta y_1, \Delta y_2, \dots, \Delta y_n)$ is given by:

$$(5) \qquad \cos\theta = \frac{1}{\rho\sigma}(dx_1\Delta y_1 + dx_2\Delta y_2 + \dots + dx_n\Delta y_n),$$

where $\rho = (dx_1^2 + dx_2^2 + \dots + dx_n^2)^{\frac{1}{2}}$ and $\sigma = (y_1^2 + y_2^2 + \dots + y_n^2)^{\frac{1}{2}}$.

To simplify the descriptions let us denote $(x_1, x_2, \dots, x_n)$ by (x) and $(y_1, y_2, \dots, y_n)$ by (y), and similarly for the variations (2) and (3).

The angle θ generally depends on tue values of (x) and (dx). If, however, θ is acute for any infinitesimal changes (dx) at a point $(a_1, a_2, \dots, a_n)$ or (a), (y) will be said to make the positive variation with respect to (x) at (a). Conversely, if θ is always obtuse, (y) makes the negative variation with respect to (x) at (a). If (y) always keeps the positive (or negative) relations with (x) at every point of the space in question, (y) will be said to make the positive (or negative) variations with respect to (x) in that space. If, however, θ becomes acute or obtuse depending upon the values of (dx), the variations of (y) with respect to (x) is indeterminate at point (a).

If θ is acute, $\cos\theta$ is positive, and if θ is obtuse, $\cos\theta$ is negative. Since the denominator of the right hand side of (5) is always positive, the positive and negative relations of the variations are determined by the following conditions:

the condition for positive variation: $\sum_i dx_i \Delta y_i > 0 \quad (i = 1, 2, \dots, n)$

the condition for positive variation: $\sum_i dx_i \Delta y_i < 0$.

3. *Positive and Negative Variations, Continued.*

The first differentials of the functions are

$$(7) \qquad dy_i = \sum_j f_{ij} dx_j \qquad (i = 1, 2, \dots, n),$$

where $f_{ij} = \partial f_i / \partial x_j$. And those at (a) may be written as:

$$(8) \qquad dy_i = \sum_j (f_{ij}) dx_j \qquad (i = 1, 2, \dots, n),$$

where the symbols in parentheses represent the partial differential coefficients at point (a).

If those differential coefficients at (a) are not all zero, the product-sums

$$(9) \quad dx_1 \Delta y_1 + dx_2 \Delta y_2 + \cdots + dx_n \Delta y_n \text{ and } dx_1 dy_1 + dx_2 dy_2 + \cdots + dx_n dy_n$$

have the same signs.

Hence, the sign of the variation at (a) can be determined accordingly as:

$$(10) \qquad \sum_i dx_i dy_i > 0 \quad \text{or} \quad \sum_i dx_i dy_i < 0. \qquad (i = 1, 2, \dots, n)$$

Substituting differentials (8) into product-sum (9), we have

$$(11) \qquad \sum_i dx_i dy_i = \sum_{i,j} (f_{ij}) dx_i dx_j \qquad (i, j = 1, 2, \dots, n).$$

Defining α_{ij} by

$$(12) \qquad \frac{1}{2}(f_{ij} + f_{ji}) = \alpha_{ij},$$

we have

$$(13) \qquad \sum_i dx_i dy_i = \sum_{i,j} \alpha_{ij} dx_i dx_j \qquad \left.\begin{matrix} i \\ j \end{matrix}\right\} = 1, 2, \dots, n.$$

The right hand side of this equation is a quadratic form in (dx). Hence the sign of the variations is determined by the positive-definiteness or the negative-definiteness of the form. If the quadratic form is positive-definite, its characteristic equation:

$$(14) \qquad |\alpha_{ij}-\lambda e_{ij}|=0 \qquad (i, j=1, 2, \dots, n;\ e_{ij}=0 \text{ for } i\neq j,\ e_{ij}=1 \text{ for } i=j)$$

has the roots which are all real and positive. The necessary and sufficient condition for this is that all the principal minor determinants of the following determinant are positive:

$$(15) \qquad D=|\alpha_{ij}| \qquad (i, j=1, 2, \dots, n).$$

In other words, the conditions of positive variations are $D_m>0$ $(m=1, 2, \dots, n)$, where $D_m=|\alpha_{ij}|$ $(i, j=1, 2, \dots, m)$.

If the quadratic form (13) is negative-definite, all the characteristic roots of (14) are real and negative. The necessary and sufficient condition for that is: $(-1)^m D_m>0$ $(m=1, 2, \dots, n)$ which is also the condition for negative variations.

4. *Invariance of Positive and Negative Variations.*

Proposition 1: The positive and negative relations of the variations are invariant for the same linear orthogonal transformations of variables and functions.

Proof: Let the linear orthogonal transformations be

$$x_i=\sum_j a_{ij}\xi_j,\ y_i=\sum_j a_{ij}\eta_j \qquad (i, j=1, 2, \dots, n),$$

where $$\sum_i a_{ij}a_{ik}=e_{jk} \qquad (e_{jk}=1 \text{ for } j=k,\ e_{jk}=0 \text{ for } j\neq k).$$

Then, $$dx_i=\sum_j a_{ij}d\xi_j,\ dy_i=\sum_k a_{ik}d\eta_k.$$

Hence, $$\sum_i dx_i dy_i=\sum a_{ij}a_{ik}d\xi_j d\eta_k=\sum e_{jk}d\xi_j d\eta_k=\sum d\xi_j d\eta_j.$$

Thus we know that the variations of (η) with respect to (ξ) has the same sign as that of the variations of (y) with respect to (x).

Proposition 2: The positive and negative relations of the variations are invariant for the following linear transformations:

$$\xi_i = \sum_j a_{ij} x_j , \; y_i = \sum_j a_{ji} \eta_j , \qquad (i = 1, 2, \dots , n),$$

where the determinat $|a_{ij}| \neq 0$.

Proof: Let (b_{ij}) be the inverse matrix of (a_{ij}), and

$$dx_i = \sum_j b_{ij} d\xi_j , \quad dy_i = \sum_k a_{ki} d\eta_k ,$$

then $$\sum dy_i dx_i = \sum a_{ki} b_{ij} d\eta_k d\xi_j = \sum e_{kj} d\eta_k d\xi_j = \sum d\eta_j d\xi_j .$$

Thus we know that the variation of (y) with respect to (x) has the same sign as that of (η) with respect to (ξ).

5. *Positive and Negative Matrices.*

Denote a matrix of order n by:

(16) $$(a_{ij}) \qquad (i, j = 1, 2, \dots n)$$

and put $$\alpha_{ij} = 1/2(a_{ij} + a_{ji})$$

then we get the symetric matrix of order n

(17) $$(\alpha_{ij}) \qquad (i, j = 1, 2, \dots , n) .$$

Its characteristic roots are all real. If they are all positive, matrix (16) will be called positive-definite. If all the roots are negative, it is called negative-definite. Otherwise it is indeterminate.

Take a linear transformation corresponding to matrix (16):

(18) $$y_i = \sum a_{ij} x_j \qquad (i, j = 1, 2, \dots , n) ,$$

multiply both sides by x_i and add them over i. Then we have

(19) $$\sum_i x_i y_i = \sum_{i,j} a_{ij} x_i x_j = \sum_{i,j} \alpha_{ij} x_i x_j .$$

If, therefore, matrix (16) is positive-definite (or negative-definite), the left hand side of (19) is always positive (or negative), whatever the values $(x_1, x_2, \dots)$ may be. In this case, the transformation (18) is called positive (or negative). If the transformation is poitive (or negative), the variation of (y) with respect to (x) is positive (or negative) at any point. Using the definitions mentioned above, the results of section 3 may be summarized as follows. If matrix (f_{ij}), $(i, j = 1, 2, \dots , n)$ is positive-definite, the variation of (y) with respect to (x) is positive at point (a). If, however, the matrix is negative-definite, the variation is negative; and if the matrix is indeterminate, so is the variation. The positive and negative relations of the variations in the defined area may similarly be discussed. It may be noted here that if a matrix (a_{ij}) is positive-definite (or negative-definite), its inverse matrix is also positive-definite (or negative-definite).

II. THE TRANSITIVITY OF POSITIVE AND NEGATIVE VARIATIONS

6. *Some Examples*:

Let $(y_1, y_2, \dots, y_n)$ be the functions of $(x_1, x_2, \dots, x_n)$ and $(z_1, z_2, \dots, z_n)$ the functions of $(y_1, y_2, \dots, y_n)$, and $(z_1, z_2, \dots, z_n)$ are the functions of $(x_1, x_2, \dots, x_n)$. Then we may consider the positive and negative relations in the variations of (z) with respect to (x). Even if (z) makes a positive variation with respect to (y) and (y) makes also a positive variation with respect to (x), (z) does not necessarily make a positive variation with respect to (x).

Example 1. Take the two linear transformations:

$$\text{(i)} \quad \begin{cases} z_1 = 2y_1 \\ z_2 = 2y_1 + y_2 \end{cases} \qquad\qquad \text{(ii)} \quad \begin{cases} y_1 = x_1 + ax_2 \\ y_2 = 2x_2 \end{cases}$$

Substitute the latter to the former and we get:

$$\text{(iii)} \quad \begin{cases} z_1 = 2x_1 + 2ax_2 \\ z_2 = 2y_1 + (2a+2)x_2 \, . \end{cases}$$

A symmetric matrix corresponding to the coefficients in the transformation (i) has the determinant:

$$\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1 > 0 \, .$$

Hence the transformation is positive. The determinant of the symmetric matrix corresponding to the transformation (ii) is:

$$\begin{vmatrix} 1 & \frac{a}{2} \\ \frac{a}{2} & 2 \end{vmatrix} = 2 - \frac{a^2}{4} \, ,$$

whose value is positive if $-2\sqrt{2} < a < 2\sqrt{2}$.

And then the transformation is positive. The determinant of a symmetric matrix corresponding to (iii) is:

$$\begin{vmatrix} 2 & a+1 \\ a+1 & 2a+2 \end{vmatrix} = 4(a+1)-(a+1)^2 = (1+a)(3-a).$$

Hence the transformation is positive if $-1 < a < 3$.
But both (ii) and (iii) can be positive only when $-1 < a < 2\sqrt{2}$. If $2\sqrt{2} \leqq a < 3$, the transformation (ii) is indeterminate. Also, if $-2\sqrt{2} < a \leqq -1$, the transformations (ii) is positive, but (iii) is indeterminate.

Example 2. Take the following two transformations, and substitute (ii) into (i) to get (iii).

$$\text{(i)} \quad \begin{cases} z_1 = 2y_1 \\ z_2 = 2y_1 + y_2 \end{cases} \qquad \text{(ii)} \quad \begin{cases} y_1 = ax_1 + 2x_2 \\ y_2 = ax_2 \end{cases}$$

$$\text{(iii)} \quad \begin{cases} z_1 = 2ax_1 + 4x_2 \\ z_2 = 2ax_1 + (a+4)x_2 \end{cases}$$

Denote by Δ the determinat of the symmetric matrix corresponding to transformation (ii), then $\Delta = \begin{vmatrix} a & 1 \\ 1 & a \end{vmatrix} = a^2 - 1 = (a+1)(a-1)$.
The conditions for (ii) to be positive are: $a > 0$, $\Delta > 0$ so that $a > 1$.
The conditions to be negative are: $a < 0$, $\Delta > 0$, so that $a < -1$.
The determinante D of the symmetric metrix corresponding to transfromation (iii) is:

$$D = \begin{vmatrix} 2a & a+2 \\ a+2 & a+4 \end{vmatrix} = a^2 + 4a - 4 = (a+2)^2 - 8.$$

If, therefore, $a > 2(\sqrt{2}-1)$ or $a < -2(\sqrt{2}+1)$, it is positive; and if $-2(\sqrt{2}+1) < a < 2(\sqrt{2}-1)$, it is negative. The conditions for (iii) to be positive are: $a > 0$, $D > 0$ so that $a > 2(\sqrt{2}-1)$; and the conditions for negativeness are: $a < 0$, $D > 0$ so that $a < -2(\sqrt{2}+1)$. As the result we obtain the following table:

Transformation	(i)	(ii)	(iii)
$a > 1$	+	+	+
$1 \geqq a > 2(\sqrt{2}-1)$	+	?	+
$a < -2(\sqrt{2}+1)$	+	−	−
$-2(\sqrt{2}+1) \leqq a < -1$	+	+	?

7. *Theorem*: Let (a_{ij}), (b_{ij}) be two matrices, and $(b_{ij})(a_{ij})=(c_{ij})$. If (a_{ij}) and (b_{ij}) are both definite (positive or negative) and (c_{ij}) is symmetric, (c_{ij}) is also positive-definite or negative-definite, as shown on the following table:

(b_{ij})	(a_{ij})	(c_{ij})
+	+	+
+	−	−
−	+	−
−	−	+

Proof: Since (c_{ij}) is symmetric, its characteristic roots are all real. Take any one of them $\bar{\lambda}$, and let the inverse of (a_{ij}) be (r_{ij}), then we have $(c_{ij}-\bar{\lambda}e_{ij})=(b_{ij}-\bar{\lambda}r_{ij})(a_{ij})$.
Therefore, $|c_{ji}-\bar{\lambda}e_{ij}|=|b_{ij}-\bar{\lambda}r_{ij}|\cdot|a_{ij}|$. Hence, $|b_{ij}-\bar{\lambda}r_{ij}|=0$.
Corresponding to the determinant of the left hand side of this equation, make a quadratic form:

$$F=\sum(b_{ij}-\bar{\lambda}r_{ij})\xi_i\xi_j\,,$$

then we know the value of its determinant is 0. Since all the elements are real, the value of F may be made equal to 0 by giving proper real numbers (not all zero) to the variables $(\xi_1, \xi_2, \dots, \xi_n)$. Suppose that such values are given and put $\xi=\sum b_{ij}\xi_i\xi_j$, $\eta=\sum r_{ij}\xi_i\xi_j$,
then we have: $\xi-\bar{\lambda}\eta=0$. By assumption (b_{ij}) is positive-definite (or negative-definite) and (r_{ij}) is the inverse of definite matrix (a_{ij}), so that ξ and η are also positive-definite or negative-definite. Hence ξ and η can never be equal to 0, and its signs are the same as those of (b_{ij}) and (a_{ij}). Thus we know the following result regarding the signs of $\bar{\lambda}$.

(b_{ij})	(a_{ij})	$\bar{\lambda}$
+	+	+
+	−	−
−	+	−
−	−	+

Here $\bar{\lambda}$ was an arbitrary root. This completes the proof.

Corollary: If either (a_{ij}) or (b_{ij}) is symmetric, and the other two matrices are positive-definite (or negative-definite), the symmetric matrix is positive (or negative-definite), and its positive and negative relations are exactly the same as indicated by the table in the theorem.

Proof: Let (b_{ij}) be symmetric and (r_{ij}) be the inverse of (a_{ij}), and we know, from $(b_{ij})(a_{ij})=(c_{ij})$, that $(b_{ij})=(c_{ij})(r_{ij})$. Applying the theorem to this equation we can easily prove this corollary.

The theorem and corollary mentioned here will be useful in the examination of stability conditions of the market equilibrium.

III. STABILITY OF DEMAND AND SUPPLY

8. *The Laws of Demand and Supply.*

Let the prices of any commodities $X_1, X_2, \dots, X_n$ be $x_1, x_2, \dots, x_n$, and the demands of those commodities $D_1, D_2, \dots, D_n$, and the demands are the functions of the prices. When the prices are (a), if the variations of (D) are negative with respect to (x), we say that the law of demand holds for (a). The condition for this is that the matrix whose elements are the partial differential coefficients of demand functions $(\partial D_i/\partial x_j)$ is negative-definite. Here we assume that the differential coefficients are not all zero. Let $S_1, S_2, \dots, S_n$, be the supply functions corresponding to the prices $x_1, x_2, \dots, x_n$. If the variations of (S) are positive for the infinitesimal changes of (x) at (a), we call that the law of supply holds. The condition for this is that a matrix $(\partial S_i/\partial x_j)$ is positive-definite. The laws of demand and supply in a certain area may similarly be defined.

9. *Stability of Demand and Supply.*

Let us define the excess demand by $y_1, y_2, \dots, y_n$ so that $y_i = D_i - S_i$. If the variations of (y) with regards to (x) in (a) are negative, the demand and supply are called stable. The condition for this is that the matrix $(\partial y_i/\partial x_j)$ is negative-definite, (see Section 5). Denote $\alpha_{ij} = \frac{1}{2}\left(\frac{\partial y_i}{\partial x_j}+\frac{\partial y_j}{\partial x_i}\right)$, and the condition for the matrix $(\partial y_i/\partial x_j)$ to be negative-definite is:

$$(-1)^m D_m > 0 \quad (m=1, 2, \dots, n) \text{ where } D_m = |\alpha_{ij}| \quad (i, j = 1, 2, \dots, m).$$

10. *Invariance of Stability in the Demand and Supply of Commposite Commodities.*

The above mentioned stability conditions are invariant for the following linear transformations which make the composite commodities out of original commodities:

$$\xi_i = \sum_j a_{ij} x_j, \quad y_j = \sum_i a_{ij} \eta_i,$$

where ξ_i and η_i are the prices and quantities of the composite commodities.

IV. THE MOVEMENT OF THE PRICE SYSTEM

11. *The Movement of Price System.*

In dynamic theory we must investigate how the price system moves from or to the equilibrium values. We shall use the terms of geometry to simplify the descriptions and understanding. We shall discuss the matter in the Euclidian space where the line segment ds is defined by $ds^2 = \sum dx_i^2$ $(i = 1, 2, \dots, n)$ and the movement of prices as the movement of a point in this space. Let us call the degree of variation in the prices with respect to time or a parameter the speed of the moving point. The acceleration of the point may be defined in the same way. That is, they are the vectors whose components are (dx_i/ds) and (d^2x_i/ds^2) respectively. In order to consider the moving point P with regard to the equilibrium point C, two methods of approach must be differentiated. In one case we may consider the speed of P; and in another we may pay attention to its acceleration.

12. *The Speed of the Moving Price System.*

Let the equilibrium point be denoted by C $(\gamma_1, \gamma_2, \cdots, \gamma_n)$ and the moving point by P $(x_1, x_2, \dots, x_n)$, and consider a vector CP and another vector whose components are (dx_i/ds), then the angle θ between the two vectors is given by

$$\cos\theta = \frac{1}{\rho\sigma} \sum_i (x_i - \gamma_i) \frac{dx_i}{ds}.$$

where $\rho = \{\sum (x_i - \gamma_i)^2\}^{\frac{1}{2}}$ and $\sigma = \{\sum \left(\frac{dx_i}{ds}\right)^2\}^{\frac{1}{2}}$.

Since $\rho\sigma$ is positive, if θ is acute, (20.) $$\sum(x_i-\gamma_i)\frac{dx_i}{ds}>0\,,$$

and if θ is obtuse,

(21.) $$\sum(x_i-\gamma_i)\frac{dx_i}{ds}<0\,.$$

On the other hand

(22.) $$\frac{1}{2}\frac{d\rho^2}{ds}=\sum(x_i-\gamma_i)\frac{dx_i}{ds}\,.$$

Hence when θ is acute: that is, inequality (20) holds, P moves away from C as s increases. If, however, θ is obtuse: that is, inequality (21) holds, P approaches to C.

13. *The Acceleration of the Moving Price System.*

Similarly considering a vector CP and another vector whose components are (d^2x_i/ds^2), we may define the angle between them by θ in the same way. If θ is acute:

(23.) $$\sum(x_i-\gamma_i)\frac{d^2x_i}{ds^2}>0\,,$$ and if θ is obtuse,

(24.) $$\sum(x_i-\gamma_i)\frac{d^2x_i}{ds^2}<0\,.$$

In other words, if inequality (23) holds, we may say that there is a force working to make P run away from C. Conversely if inequality (24) holds, there is a force working to make P move toward C.

Leon Walras, when he said, while quoting an example of the pendulum, that there is a force working to pull back the moving price system toward the equilibrium point, had in mind the way of approach in this section, and not in the last section.

Even if there always is a force working to pull P back to C for any value of s, P does not necessarily approach to C as s increases. It is clear, therefore, that the stability conditions for the variation of the price system cannot be discussed by the Walrasian condition only. Conversely, if there is a force working to make P diverge from C, P does not necessarily run away from C as s increases; on the contrary, it may approach to C. Here are relevant initial conditions.

14. *The Relations Between the Acceleration of the Changing Prices and the Excess Demands.*

Let the excess demand functions be $f_1, f_2, \dots, f_n$, and denote a vector whose components are $f_1, f_2, \dots, f_n$, by f; that is, $f=(f_i)$ and the acceleration by a; that is, $a=(d^2x_i/ds^2)$. Then the angle θ between these two vectors is given by

$$\cos\theta=\frac{1}{\rho\sigma}\left(\sum f_i\frac{d^2x_i}{ds^2}\right) \qquad (i=1, 2, \dots, n)$$

where

$$\rho=(\sum f_i^2)^{\frac{1}{2}}, \qquad \text{and} \qquad \sigma=\{\sum(d^2x_i/ds^2)^2\}^{\frac{1}{2}}.$$

Since $\rho>0$, $\sigma>0$, if θ is acute,

$$\sum f_i\frac{d^2x_i}{ds^2}>0, \tag{25.}$$

if θ is obtuse,

$$\sum f_i\frac{d^2x_i}{ds^2}<0. \tag{26.}$$

It may be noted that in the case of a single commodity the positive or negative relations between the excess demand and the price variations may be expressed by $f\frac{d^2x}{ds^2}>0$ or <0. If the relation is positive, the excess demand produces a force to raise the price, and the excess supply lowers the price. If the relation is negative, the converse holds.

15. *Stability Conditions of the Market Equilibrium in Dynamic Theory.*

It is of utmost importance to clarify the stability conditions of market equilibrium mentioned in Section 9 and the stability conditions of price variations in dynamic theory. We must, however, satisfy ourselves here by sketching part of the results of our analysis of this problem. In the following discussion we shall take time t as our parameter. In dynamic theory we may consider a point with no acceleration; that is, $d^2x_i/dt^2=0$ $(i=1, 2, \dots, n)$, and call it the center. The center is not necessarily the equilibrium point; in other words, the center does not necessarily satisfy the conditions: $f_i=0$ $(i=1, 2, \dots, n)$. We shall assume here that the center is the equilibrium point and represents it by $(\gamma_1, \gamma_2, \dots, \gamma_n)$. Next, we use the following notations:

$$F=\sum_j (x_j-\gamma_j)\frac{d^2x_j}{dt^2}, \quad G=\sum_j df_j dx_j, \quad \text{and} \quad H=\sum_j f_j \frac{d^2x_j}{dt^2}.$$

From the previous discussions we know that in (γ) there is a force working to pull back the price system to the equilibrium point if $F<0$ and a force works to make the price system diverge from C if $F>0$. (see Section 13). If $G<0$, the demand and supply have stability with respect to the prices and if $G>0$ they are unstable, (see Section 9). If $H<0$, the excess demand and the acceleration of the price variations move in the opposite directions, and if $H>0$, they move in the same direction, (see Section 14).

Assume that the signs of H and F do not change in the neighborhood of the equilibrium point. Then we have the following relationships:

$$(27.) \qquad FGH>0.$$

This may be expressed by the following table:

	F	G	H
I	−	−	+
II	−	+	−
III	+	+	+
IV	+	−	−

A brief explanation of the table may be added for case I. In that case, there is a force working to pull back the price system to the equilibrium point —the Walras conditions— and the demand and supply are stable for the price variations. The table shows that the excess demands in this case move in the same direction as the acceleration of the prices. In Case IV, demand and supply are stable for the price variations, but there is a force working to diverge the price system from the equilibrium point, so that the excess demand moves in the opposite direction to the acceleration.

If we consider the relations between the excess demands and the acceleration of the price variations, it would be appropriate to suppose that Case I is the most stable. In other words, among the afore-stated three inequalities, we may take any two conditions and thereby define the stability conditions. It should be noted that the three inequalities are mutually independent, but any one of them may be obtained from the other two as inequality (27) holds. Hence, it is not sufficient to have either Walras' conditions or the stability conditions of demand and supply in order to define stability of market equilibrium. The stability conditions must be given by the two conditions among those inequalities mentioned above. (October, 1951)

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A PROBLEM OF SMALL AND MEDIUM-SIZED BUSINESSES IN JAPAN

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I

The enterprise its plant, or its establishment, each is a compound of its constituent elements, production-factors and products. Therefore the size of enterprise or plant is generally measured by the magnitude or amount of these elements. Here one or sometimes many (combined) of them are chosen as the measure of business size. On the other hand, another standard can be used for demarcating one business of a certain size and the other of larger one. This standard is a degree of business efficiency which is different according to the size of firm. In this case the degree of business efficiency is considered as depending on some conditions. That is to say: firstly it depends on the manner in which a certain number of production-factors is combined, secondly on the particular feature of structure which this combination builds in the firm concerned. Therefore, if the former standard for measuring the size of business is called quantitative, the latter is qualitative.

Of these two standards for measuring business size, we should like first to investigate the quantitative, while on the qualitative we will touch later. As regards the former standard, many writers adopt as the measure of size the amount of assets, sales, income, capital or employees of enterprise or its plant. There exists a relation between any of these above indices which can be used as a measure of business size. This relation can be best understood, if we refer to Marx's capital-value circulation formula of individual capital in this respect. This formula is as follows:

$$G-W\begin{cases}A\\Pm\end{cases}\cdots\cdots\cdots P\cdots\cdots\cdots W'-G'\begin{cases}G\\g\end{cases}$$

G = amount of money-capital,
W = value of commodity put-in (A and Pm),
A = value of labour,
Pm = value of means of production—raw material, material and equipment,

P = production,
W' = value of commodity produced,
G' = proceeds of commodity produced,
g = surplus value of production.

Now let us compare these items in Marx's formula to various measures as mentioned above. Then we can know the following correspondences, that is,

G – capital,
A – number of employees in terms of wages,
Pm – a part of asset,
W' – other part of asset,
G' – proceeds,
g – income.

According to Marx, production means a circulation of value of capital invested. Therefore, we think, if G (the amount of money-capital) represents the size of business, W (the value of put-in commodity), W' (the value of commodity produced) and G' (its proceeds) have equally to do so. Each one of these items has to express the same size of business as the other, so far as the value of each item circulates to its full extent. Moreover, on a given stage of technical development and under a certain economic circumstance, A (the value of labour) and g (the surplus value of production) might be correlated to G and perhaps be expressed in a certain ratio of this. In short, for measurement of business size it may likely be the same which of these items, —G, W, W', g or the like— is adopted as a standard.

Theoretically speaking, the size of enterprise or plant which any of these measures respectively determines should not be different from that ascertained by other measures. For each measure is an index which ought to express the size of business. However, in actual cases the size of business can be quite different according to adopted measures. This is partly due to the fact that the datum referring to these items as stated above is not complete enough to suit the measurement. Furthermore, a size measured by one item is not equivalent to that by others even theoretically. An example of this fact was introduced by Mr. M. A. Adelman in his treatise, "The Measurement of Industrial Concentration." He said, among other things: "Sales provide the most easily available measure of size, but the weakest, in that they disregard the extent of vertical integration. Thus, two firms may each make ten per cent of an industry's sales; but if one merely purchases all the components, adds 'a lick and a promise,' and resells, it is obviously much smaller than the other, which undertakes all or much of the whole productive process."[1] Although the sales of two firms may be the same, the assets, employees and income of these firms are different according to whether they are vertically integrated or not. Apart from this example, the value of each production-factor or product does not necessarily circulate to its full extent. This is one of the fundamental reasons why the size of business is differently measured according to the index adopted.

Thereupon, some writers have placed much emphasis on providing two or more measurements combined in this respect, whether for theoretical or for practical reasons. They often argue for combining the number of employees and the amount of assets as a standard. Then, there arises a question in what way these two elements can be combined.

There is another important feature in choosing the measure of business size. That is a requirement for invariability in the value of the standard. In this respect neither asset, income, sales nor capital seems completely to satisfy this demand, because general price level affects the value of them. Consequently some resort to standards of physical nature, for instance, number of employees.

If the above two considerations be taken account of, there may be provided as a measurement the number of labourers and the capacity of operating machinery combined. Here the capacity of operating machinery plays a role as a representation of all other fixed assets. Furthermore this measurement assumes a particular production function which exists between the number of labourers and capacity of machinery on the one hand and the volume of products on the other. This idea is similar to Douglas' production function. However, this measurement involves some defects in such a respect as it does not pay due attention to the efficiency of labour as well as to that of machinery. These defects can be, to some extent and in particular cases, adjusted by giving weights to labour or machinery according to their respective efficiencies. Anyhow, if the type of this particular production function is determined, a measure of business size can be found accordingly. Let us take Douglas' production function as an instance and apply this to our analysis, although in some modified form.

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[1] Adelman, M. A., The Measurement of Industrial Concentration, *The Review of Economics and Statistics*, November 1951, p. 272.

That is,

$$P = xL^{y}C^{z}\ ,$$

where $P=$ index of amount of manufactures. These manufactures are a kind of value-added product. That is: their amount is recalculated in such a way as the value of raw material, material or the like used is deducted from that of manufactures,

$L=$ index of number of employees, the number being adjusted according to the employee's efficiency,

$C=$ index of capacity of operating machinery, the capacity being adjusted according to the machinery's efficiency.

Then,

$$\log P = \log x + y \log L + z \log C,$$

to determine the values of x, y and z, we form three equations. In this case P_i, L_i and C_i $(i=1,\ 2,\ 3)$ are parameters, and we choose such values of them as those at periods when economic conditions are stable.

$$\text{If}\quad \begin{vmatrix} 1 & \log L_1 & \log C_1 \\ 1 & \log L_2 & \log C_2 \\ 1 & \log L_3 & \log C_3 \end{vmatrix} \neq 0;$$

we can determine the values of x, y and z.

Now, by these values, x_0, y_0 and z_0, we can express the size of business in such a formula as

$$x_0 L^{y_0} C^{z_0}, \text{ where } L \text{ and } C \text{ are given.}$$

Lastly we ought to note that the measure of business size may be differently provided according to the branches of trade and industry. That is : one measure is suitable to a certain branch, but not to the other.

II

As every one knows, large scale business has many advantages and also some disadvantages. So it is with the small and medium-sized. This leads to an examination as to the limitation which is imposed on the size of business to realize its profit. Such limitations are of technical as well as of economic nature. In this respect, one important problem is to find the optimum size of business. For the optimum size of business can be said to denote a limitation on its scale. However, there is no common definition of this optimum size among writers. Usually economists measure the optimum size of business by means of its fixed asset or equipment. However there are two different views in this respect. Some of these economists understand the optimum size as the business size of which the average cost of production is lower than that of any other size. On the other hand, others mean by it such a business scale as can attain the highest profit or profit-rate among any conceivable business sizes. Whatever the definition of optimum business sizes may be, suppose that the optimum size of business in a certain branch of trade and industry is determined theoretically as well as practically. Then, there ought to be no question concerning the scale of business. The problem as to whether small, medium-sized or large business is to be preferred ceases to exist in the said branch.

Aside from the optimum business size or limitations on production scale, the entrepreneur must very often face the question whether it is profitable to extend his establishment. The consideration to answer this question is the possibility of reducing the average cost of production or of increasing the rate of profit by the extension. These possibilities can be realized only when, upon the enlargement of the establishment, either the rate of production increase exceeds that of equipment extension or average variable cost decreases. Let us explain this relation by mathematical analysis.

We should like to begin with the reduction of average cost of production. For this purpose let us take advantage of that famous but simple formula of cost of production by K. Bücher. Bücher introduced this formula in his explanation of mass-production.

That is :

$$\text{Average cost of production } K=\frac{C}{x}+V,$$

(for the sake of convenience we use both the symbol and its meaning somewhat differently than in the original formula)

where

$C=$ total invariable cost, consisting simply of the value of equipment,

$V=$ average variable cost,

$x=$ amount of production.

Now we put an original average cost of production as

$$K_1=\frac{C}{x}+V.$$

Further we take an average cost of production when the equipment is enlarged as

$$K_2=\frac{C+\Delta C}{x+\Delta x}+V+\Delta V, \qquad \text{where } \Delta C>0,\ \Delta x\gtrless 0.$$

Then the condition that

$$K_1-K_2>0 \qquad \text{is}$$

$$K_1-K_2=\frac{C}{x}+V-\left(\frac{C+\Delta C}{x+\Delta x}+V+\Delta V\right)$$

$$=\frac{C\Delta x-x\Delta C-x^2\Delta V-x\Delta x\Delta V}{x(x+\Delta x)}>0,$$

$$\therefore\quad C\Delta x-x\Delta C-x^2\Delta V-x\Delta x\Delta V>0\,.$$

Now,

a) if $\Delta x=0$,

$$-x(\Delta C+x\Delta V)>0, \qquad \therefore \frac{\Delta C}{x}<-\Delta V,$$

$$\frac{\Delta C}{x}>0, \qquad \Delta V<0,$$

this means that if x does not increase, V decreases,

b) if $\Delta V=0$,

$$C\Delta x-x\Delta C>0, \qquad \frac{\Delta x}{x}>\frac{\Delta C}{C},$$

this means that the rate of x's increase is larger than that of C's increase,

c) if $\Delta x=0$, $\Delta V=0$,

$$-x\Delta C>0, \qquad \therefore \Delta C<0,$$

this is contrary to the assumption.

Now we will investigate this problem from the viewpoint of increasing the profit rate by the extension of establishment. This means that we will compare the profit rate of the original establishment to that of the extended.

Then, the rate of profit

$$G=\frac{px-\{C+V(x)\}}{I} \qquad (1),$$

where $p=$ price of product $=$ constant (we assume that free competition prevails for small and medium-sized business),

$x=$ amount of product,

$C=$ amount of invariable cost, mostly being the depreciation in the value of equipment mainly due to its use for producing x,

$V(x)=$ variable cost,

$I=$ amount of the entrepreneur's own capital invested in the firm.

Now we assume that all of I is used for the purchase or construction of equipment, and further we assume

$$C = \frac{I}{\alpha}, \qquad \alpha = \text{constant},$$

meaning that the ratio of depreciation in the value of equipment is constant.

Therefore, (1) becomes following formula,

$$G = \frac{px - V(x)}{I} - \frac{1}{\alpha} \qquad (2),$$

(2) denotes a certain form of function, where I is a parameter and x is an independent variable. Thus,

$$\frac{px - V(x)}{I} - \frac{1}{\alpha} = f(x, I).$$

The profit rate of the original equipment

$$G_1 = \frac{px - V(x)}{I} - \frac{1}{\alpha} \qquad (3).$$

The profit rate of the enlarged equipment

$$G_2 = \frac{p(x + \Delta x) - \{V(x) + \Delta V(x)\}}{I + \Delta I} - \frac{1}{\alpha} \qquad (4).$$

Now if $G_2 - G_1 > 0$,

$$\frac{p(x+\Delta x)}{I+\Delta I} - \frac{V(x) + \Delta V(x)}{I + \Delta I} > \frac{px}{I} - \frac{V(x)}{I} \qquad (5).$$

Let us consider several cases for proving (5). Then we can know five cases as follows:

i. $$\frac{p(x+\Delta x)}{I+\Delta I} > \frac{px}{I}, \qquad \frac{V(x)+\Delta V(x)}{I+\Delta I} = \frac{V(x)}{I},$$

ii. $$\frac{p(x+\Delta x)}{I+\Delta I} = \frac{px}{I}, \qquad \frac{V(x)+\Delta V(x)}{I+\Delta I} < \frac{V(x)}{I},$$

iii. $$\frac{p(x+\Delta x)}{I+\Delta I} > \frac{px}{I}, \qquad \frac{V(x)+\Delta V(x)}{I+\Delta I} < \frac{V(x)}{I},$$

iv. $$\frac{p(x+\Delta x)}{I+\Delta I} < \frac{px}{I}, \qquad \frac{V(x)+\Delta V(x)}{I+\Delta I} < \frac{V(x)}{I},$$

however in this case, $\frac{px}{I} - \frac{p(x+\Delta x)}{I+\Delta I} < \frac{V(x)}{I} - \frac{V(x)+\Delta V(x)}{I+\Delta I}$,

v. $$\frac{p(x+\Delta x)}{I+\Delta I} > \frac{px}{I}, \quad \frac{V(x)+\Delta V(x)}{I+\Delta I} > \frac{V(x)}{I},$$

however in this case, $\frac{p(x+\Delta x)}{I+\Delta I} - \frac{px}{I} > \frac{V(x)+\Delta V(x)}{I+\Delta I} - \frac{V(x)}{I}$.

Thus we are comparing the profit rates of both circumstances in the light of turn over ratio of entrepreneur's own capital as well as of ratio of entrepreneur's own capital to variable cost.

Now $\frac{p(x+\Delta x)}{I+\Delta I} \gtreqless \frac{px}{I}$, where p, x, Δx, I and ΔI are all positive,

$$Ip(x+\Delta x) \gtreqless (I+\Delta I)px,$$

$$I\Delta x \gtreqless x\Delta I, \quad \frac{\Delta x}{x} \gtreqless \frac{\Delta I}{I}.$$

We put

$$\frac{\frac{\Delta x}{x}}{\frac{\Delta I}{I}} = \kappa,$$

then, $\frac{p(x+\Delta x)}{I+\Delta I} \gtreqless \frac{px}{I}$ can be expressed in such a way as $\kappa \gtreqless 1$.

In a similar way, if we put

$$\frac{\frac{\Delta V(x)}{V(x)}}{\frac{\Delta I}{I}} = \lambda,$$

then, $\frac{V(x)+\Delta V(x)}{I+\Delta I} \gtreqless \frac{V(x)}{I}$ can be expressed in such a way as $\lambda \gtreqless 1$.

Now from (5),

$$Ip\Delta x - I\Delta V(x) > px\Delta I - V(x)\Delta I,$$

$$p\frac{I\Delta x}{x\Delta I} - \frac{I\Delta x}{x\ \Delta I}\frac{\Delta V(x)}{\Delta x} > p - \frac{V(x)}{x},$$

$$p\kappa - \kappa\frac{\Delta V(x)}{\Delta x} > p - \frac{V(x)}{x},$$

$$\kappa\left\{p-\frac{\Delta V(x)}{\Delta x}\right\}>p-\frac{V(x)}{x}.$$

We assume

$$p-\frac{V(x)}{x}>0,$$

then,

$$\frac{p-\dfrac{\Delta V(x)}{\Delta x}}{p-\dfrac{V(x)}{x}}>\frac{1}{\kappa} \tag{6),}$$

if $$1\geqq\kappa>0,$$

$$\frac{p-\dfrac{\Delta V(x)}{\Delta x}}{p-\dfrac{V(x)}{x}}>1, \qquad \frac{V(x)}{x}>\frac{\Delta V(x)}{\Delta x},$$

$$\therefore \quad \frac{\dfrac{\Delta x}{x}}{\dfrac{\Delta V(x)}{V(x)}}>1.$$

Now we utilise the above analysis in the five cases as stated before, then we know the following relations.

That is

i.	$\kappa>1$	$\lambda=1$	$\mu>1$
ii.	$\kappa=1$	$\lambda<1$	$\mu>1$
iii.	$\kappa>1$	$\lambda<1$	$\mu>1$
iv.	$\kappa<1$	$\lambda<1$	$\mu>1$
v.	$\kappa>1$	$\lambda>1$	μ unknown

where

$$\kappa=\frac{\dfrac{\Delta x}{x}}{\dfrac{\Delta I}{I}}, \quad \lambda=\frac{\dfrac{\Delta V(x)}{V(x)}}{\dfrac{\Delta I}{I}}, \quad \mu=\frac{\dfrac{\Delta x}{x}}{\dfrac{\Delta V(x)}{V(x)}}=\frac{\kappa}{\lambda}.$$

Let us make an economic interpretation of this table. The conditions for increasing the profit rate of business by enlargement of equipment are as follows:

That is,

i. the rate of production increase is larger than that of equipment enlargement. The latter rate is equal to the rate of increase of variable cost. Consequently the rate of production increase is larger than the rate of increase of variable cost,

ii–iv. the rate of production increase is equal to, larger or smaller than the rate of equipment extension. However the rate of increase of variable cost is smaller than the latter and also than the rate of production increase,

v. the rate of increase of production as well as that of variable cost is larger than the rate of equipment extension. However the relation between the rate of production increase and that of increase of variable cost is unknown.

So far we have analysed, with respect to the profit rate of business, the relations between (1) the rate of production increase, (2) the rate of increase of variable cost, and (3) the rate of equipment extension. Thus we can conclude broadly that to realize larger profit rate by extension it is necessary to make either the rate of production increase larger or the rate of increase of variable cost smaller than that of equipment extension. Furthermore each of these cases generally accompanies such a relation that the rate of increase of variable cost is smaller than that of production increase. This means that average variable cost under original equipment is larger than that under the extended. Therefore if these conditions do not exist, the extension of equipment may not be desirable. This undesirableness will contribute to determining the proper fields of trade and industry which small, medium-sized or large scale respectively suits best. Also the limitations on the scale of business as a whole and its optimum size in particular play the same role.

These notions are well expressed in some examples. As probably every one admits, in the art industry and in the repairing shop, there is hardly any possibility of operating large scale business. The same thing to some extent can be said of an industry of such kind that the demand for its product is remarkably affected by the change of fashion, taste and the like. In these fields, although the production increases by the extension of equipment, the rate of its increase is smaller than that of extension. On the other hand, large scale suits shipbuilding, aircraft industries and also those industries in which the indivisibility of productive unit or process prevails. In these branches average and marginal variable costs generally decrease with the extension of establishment.

III

Actually businesses of any and every size are in a competition with each other. And this competition takes place not only among the firms of similar scale, but also between those of different sizes. However, the fundamental feature of small and medium-sized businesses consists in the fact that much emphasis ought to be placed on their competition against the large scale.

Generally speaking, the problem of small and medium-sized businesses is partly economic and partly a matter of social welfare. The abundance of these businesses is one of Japan's main characteristics; and this is due to her overpopulation and scarcity of natural resources. Whenever economic prospect is bright, small business in particular and the medium-sized to some extent grow like "mushrooms after rain." People who have some amount of capital, some technical and economic knowledge will very often start business and indeed mostly the small-sized. They usually pay little attention to the proper scale of operation in the branch concerned.

Now let us investigate the features of small business in Japan. The productivity of this business consists mainly in man-power and its skillfulness. Of course it provides tools, machines and other equipment to some extent, but they play a role only as a supplement to man-power. Therefore there takes place no substitution between labour and machinery. Small business tries to compensate for the accompanying disadvantage by low wages, in particular by labour of its owner's family. Moreover working place is at the same time dwelling, thus reducing cost of production.

On the other hand, the entrepreneur of small business boasts of the merit of his one-man control on its operation. Its advantage seems to be such a state of affairs that his direction goes through all the work in his firm. That is to say, the entrepreneur's initiative in his business operation is reflected precisely in each piece of the product, just as in every service, which his firm provides. This feature of course deserves criticism from various viewpoints. Whatever they may be, the entrepreneur will satisfy himself only if his one-man control is realized to the full extent; and this satisfaction encourages him to keep the business even under unfavourable economic circumstances.

Setting aside for the moment the judgement as to whether one-man control in small business is desirable, let us review the relation between one-man control and business size. One-man control usually accompanies line type of management, and the number of workmen under this management depends upon the extent to which one-man's decision can direct their actions. There is a natural limitation on this extent; and the number of workmen is restricted accordingly. Therefore a firm of such a sort that its productivity mainly consists in man-power and its size is restricted by the employees under one-man control can not be large. In short not-large sized business is the logical result of one-man control.

IV

Now we should like to investigate medium-sized business in Japan. Suppose an entrepreneur has a larger amount of capital to invest and perhaps wider knowledge of technology and economy than the small business man (as mentioned above). He will start bigger business and employ more labour; and then he can take advantage of division of labour. The division of labour underlies the minute study of work. On the one hand, the productive process is divided into particular smaller units as far as possible, while on the other, the labourer's skillfulness is analysed and his specialized ability is konwn. Then the whole production is arranged in such a way that each specialized worker is assigned to a particular unit of the productive process. Furthermore it involves such a relation that the former corresponds to the latter one-to-one.

The said entrepreneur may place much more emphasis on labour than on machinery for his production. Besides he may also adhere to one-man control of his firm. However in his case, the entrepreneur introduces a somewhat complicated line type of management, because employees are more numerous than in small business. That is to say, this line type consists of two-stages pyramiding control and sometimes is combined with functional type. The attached graph shows the structure of this type of managment.

As stated above, this entrepreneur has a medium amount of capital, which imposes a limitation upon the size of his business. Beside this, due to one man control he faces a restricted number of employees and therefore a limitation on business size. This feature is not by its nature different from that of small business. However in two-stages pyramiding control the coefficiency of control is smaller than in direct control. Suppose a coefficiency of direct control w, where $0 < w \leqq 1$. Then in two-stages control the coefficiency of ultimate control will be w^2. To compensate for or to avoid this sort of efficiency decrease the entrepreneur resorts either to increasing tools or machineries in the plant or to reducing the number of employees.

Thus the medium-sized business has grown in Japan. In manufacturing the medium-sized plant has some similarity to the " Manufaktur " or " Manufacture " in Marx's Kapital. " Manufaktur " is a plant in which productive power mainly consists in man-power, in particular in hand-work, in division of labour and in cooperation. Marx divided the " Manufactur " into two sorts in terms of the nature of their development. One sort arises as many independent handworkers—they were formerly engaged in making one of various parts or components of a finished product—assemble as employees under a single capitalist to complete the said commodity. Here each of these artificers does the same work as before, but passes over his own product to another's hand for the next process of manufacturing. The other sort, on the contrary, arises when a number of craftsmen— all of them having done the same kind of work— are employed by a capitalist. However now the operation is rearranged and particular work is assigned to each specialized labourer.

V

The above description merely refers to an aspect of small and medium-sized businesses in Japan. However this aspect, to whatever extent it may be, is observed in most of them.

Small and medium-sized businesses find a common feature in that both of them are managed under one-man control of the entrepreneur. Or, reversing the statement: this feature seems to determine the size of business, owing to the limited number of employees. Thus small and medium-sized businesses can be defined in terms of the number of employees. This number can not be definitely figured, but it is different according partly to the branches of trade and industry, partly to the individuality of the entrepreneur. However, so far as controlling power generally finds a limitation on its effect, the number of employees either of small-sized or of medium-sized will not be so large as commonly thought. We may say, small business has five to thirty employees, while the medium-sized has the employment of thirty to two hundred work men. This figure is perhaps much smaller than what is considered definitive for American small or medium-sized businesses.

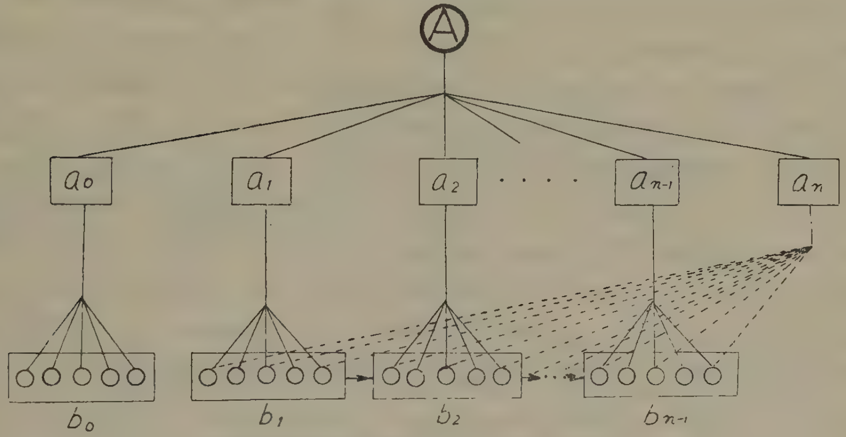
Furthermore, the number of employees, as related to the scope of the entrepreneur's controlling power, can be considered as a line demarcating business sizes from another viewpoint. The one-man control system is a peculiar structure of management and involves a particular efficiency of business operation, whether it is high or low. That is to say, small and medium-sized firms based upon one-man control, in particular handwork and "Manufaktur"-like plants, show such efficiencies as are qualitatively different from the efficiency of the large-sized. Besides, small business under one-man control on its part is qualitatively differentiated in its efficiency from the medium-sized.

As stated above, the coefficiency of controlling effect of the former is larger than that of the latter (1 : w). Thus one-man control and number of employees affected by it will be an element for giving a qualitative definition to small, medium-sized and large businesses.

There is one more point to be noted concerning one-man control. It tends easily to accompany family-like binding between entrepreneur and labour. This binding is conspicuously strong in Japan, because the influence of trade unions has not yet much prevailed among the employees of small and medium-sized firms. We often call the labour relation under this binding a " feudal system of employment." This employment mechanism deserves criticism from various points of view, whether favourable or unfavourable. It can promote co-operation between the entrepreneur and employees, but it can also play such a role as in the exploitation of labour. From the viewpoint of economic democracy it may meet the opposition of the labouring class. However one-man control accompanying such family-like binding is sometimes found also in the big enterprises of Japan, particularly in firms which have grown from small size. Then, even if the advantage of this mechanism be granted in small or medium-sized firms, it will be surely disadvantageous in big enterprises with respect both to business efficiency and to human relations.

Small and medium-sized businesses, particularly in manufacturing, sometimes make combine with large business. The former supplies finished, unfinished or component products to the latter by a long-period contract, while the latter supports the former technically as well as financially. (Here small or medium-sized business stands in a complementary position with the large-sized.) Irrespective of this relation, the market of products by small and medium-sized enterprise is usually very narrow. Even in that they confront a competition with large business. Thereupon small and medium-sized firms usually pay particular attention to the differentiation of their commodities, in order to survive their competition. And the differentiation of commodity can be very often effected by special knowledge and technology of small and medium-sized entrepreneur. In this respect one-man control seems to find its justification, for the direction of such knowledge and technology can affect precisely any and every product or service under one-man control of the entrepreneur.

A Managing Type of Production of A Medium-Sized Plant.



A: entrepreneur,
a_0: sales and accounting manager,
a_i ($i = 1, 2, \ldots, n-1$): foreman controlling the group assigned to him,
a_n: foreman supervising every group,
b_0: sales and accounting section,
b_i ($i = 1, 2, \ldots, n-1$): group of labourers in each specialized unit of the manufacturing process.

1) The number of workmen in b_i is not necessarily the same with other b_i, but is variously determined according to the nature of different works assigned to it. However each b_i generally consists of nearly n persons.
2) a_n is a specialist in particular technology and directly supervises workmen in every b_i. Sometimes A himself directs these workmen in place of a_n, when A is such a specialist.
3) Arrow indicates the flow of components produced in each b to next b. Thus a product will be finished in b_{n-1}.

NOTES ON NON-LINEAR BUSINESS CYCLE THEORIES

SHINICHI ICHIMURA

Preface

In the recent years some economists have attempted to discuss the trade cycle or economic fluctuations in terms of non-linear differential or difference equations. In spite of their esoteric appearances, what they really amount to is nothing intricate. This paper is intended in the first place to supplement some diagrams to the mathematical arguments so far offered by many economists and simplify the understanding of the essentials of non-linear theories. In the second place, the paper tries to explore the possibility of constructing a business cycle theory on the basis of the neo-classical theory of the firm. By introducing explicitly the price element into the theory, this approach will clarify the ambiguous points of old business cycle theories in the relations between the theory of the firm and business cycle theories.[1]

Part I. A Graphical Supplement

1. The non-linear macro-dynamic theories dealing with total investment, savings, and output may be classified into two categories. In the first type of theory investment is dependent on the level of output and capital stock, whereas in the second it depends on the rate of change in output—the acceleration principle. For the sake of following arguments it is convenient to discuss the kaldor theory in terms of a few diagrams. In his theory, it is assumed, the gross investment (I_t) depends

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[1] N. Kaldor, "A Model of the Trade Cycle," *The Economic Journal*, March 1940; J. R. Hicks, *A Contibution to the Theory of the Trade Cycle*, Oxford, 1950; N. Georgescu-Roegen, "Relaxation Phenomena in Linear Dynamic Models," in *Activity Analysis of Production and Allocation*, edited by T. C. Koopmans, New York, 1951; T. Yasui, "Self-Excited Oscillations and Business Cycles," *Riron-Keizai-Gaku*, November 1952; R. M. Goodwin, "The Non-Linear Accelerator and the Persistence of Business Cycles," *Econometrica*, January 1951; M. Morishima, "Three Models of Non-Linear Business Cycle Theories," *Riron-Keizai-Gaku*, November 1953; S. Ichimura, "Toward a General Non-Linear Macro-Dynamic Theory of Economic Fluctuations," in *The Post Keynesian Economics*, The Rutgers University Press, 1954.

positively on the level of output (Y_t) and negatively on the amount of existing capital stock (K_t). For simplicity let us assume that the gross investment function can be written as:

$$I_t = f(Y_t) - mK_t\,, \tag{1}$$

where $f(Y_t)$ is a non-linear function of Y_t as is shown in Fig. 1, and m is a positive constant. In Fig. 1, I_t ($t = 0, 1, 2$) represents the investment function for different quantities of capital equipment such that K_0, K_1, K_2. As for the saving function (S_t) and the replacement investment function (R_t), it would be permissible to assume that S_t is proportional to Y_t, and that R_t is a linear function of Y_t but a constant depreciation per unit period exists due to the mere efflux of time:

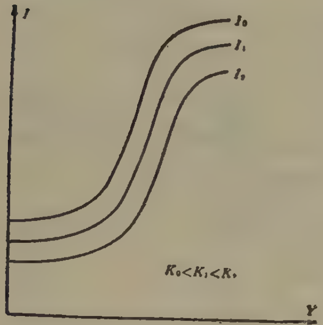

Fig. 1

$$S_t = sY_t \tag{2}$$

$$R_t = rY_t + a\,. \tag{3}$$

The two functions are shown in Fig. 2. Then we assume, according to Mr. Kaldor, that the rate of change in output is proportional to the change in output is proportional to the difference between investment and saving:

$$Y_{t+1} - Y_t = \mu(I_t - S_t)\,, \tag{4}$$

where μ is assumed for the moment to be a positive constant and may be called the "response coefficient" of supply to Effective Demand. Hence, if investment is equal to saving at time t, then income or output will not change. If, moreover, the gross investment is equal to the replacement investment, capital stock will remain unchanged. When both Y_t and K_t do not change, we may properly state that the economic system is in the long-run equilibrium. In terms of equations, this long-run equilibrium can be defined by the following two equations. And the diagrammatical representation is given by Fig. 2, in which the intersecting point of the three curves —I_t, S_t, and R_t—gives the long-run equilibrium represented by (Y^0, K^0).

$$I_t = S_t\,; \qquad I_t = R_t\,. \tag{5.1, 2}$$

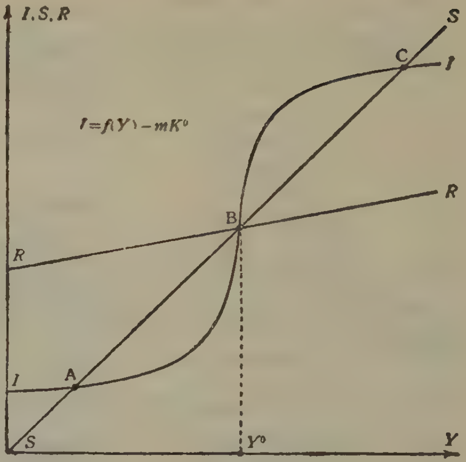

Fig. 2

In Fig. 2, B gives the long-run equilibrium value of Y_t. As far as output is concerned, A and C may seem to be the other possible equilibrium points. But at C output cannot remain unchanged because the excess of gross investment curve over replacement will shift the investment curve downwards. A similar argument will apply to A also. This means that if output is very far from the long-run equilibrium value, there is a force working to bring down the level of economic activity towards Y^0. On the other hand, however, point B is not a stable equilibrium point because in the neighborhood of B the slope of the investment curve exceeds that of the saving curve so that output tends to run away from Y^0. The interplay of these two counteracting forces may be expected to bring about a steady oscillation of output with a certain amplitude which is called a " limit cycle. " Ws shall now proceed to the graphical analysis of this cyclical behavior of output.[2]

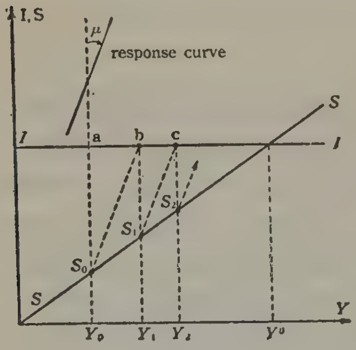

Fig. 3

3. Let us begin by reminding ourselves of the familiar diagram of the multiplier mechanism. (See Fig. 3.) Suppose that investment is independent of output and capital, and that saving is a proportional function of real income. Then, if investment increases from the level of I^0 to I^1, output or real income will be increased in the first period by the amount proportional

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[2] It may be of some economic interest to note that from (5) we obtain: $Y^\circ = a/(s-r)$; $K^\circ = f(a/s-r)/m$. Hence, the greater a or r is and the smaller s or m is, the larger will be Y° and K°. The effect of the change in the functional shaper of $f(Y)$ can easily be seen in Fig. 3. These results are interesting because (Y°, K°) roughly represents the levels of Y and K around which the cycle will take place.

to the difference between I and S. If the responsiveness of supply to demand is such that the gap of I and S is filled in one period—the response coefficient in (4) is unity—, the response curve will have the slope of 45°. As is well-known, we can get the output in the next period dy drawing a line parallel to the response curve from the original equilibrium point to the I^1 line and dropping a line from b to the abscissa. For the following periods we have only to repeat the same drawing. If, however, investment depends on Y, (See Fig. 4), the diagram is a little more complicated. In order to get the income of the first period, we have to draw two lines; one from a parallel to the abscissa and the other from S_0 parallel to the response curve. From the intersecting point of the two lines we draw a line parallel to the vertical axis, thus obtaining Y_1 which is the output in the first period. It would be obvious to obtain a series of Y_2, Y_3 by repeating semilar procedures. It should be noted that this method is applicable also even if the investment function and the response curve are nonlinear. We are now in a position to show a graphical solution of the Kaldor theory.[3]

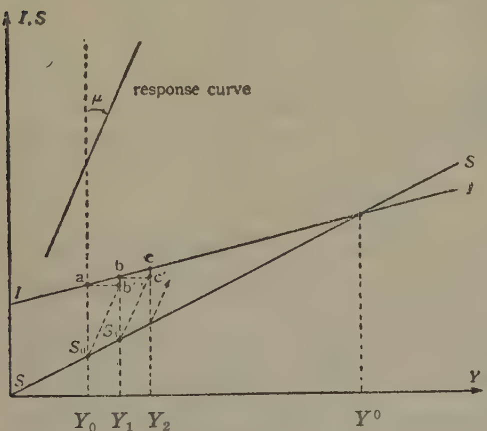

Fig. 4

The only modification to make is to introduce, as is indicated in Fig. 5, the "Kalecki curve" whose slope against the vertical axis is equal to m. As an example we shall show how to solve the Kaldor system of equations by this diagram when Y_0 is given at the level indicated in Fig. 5 and K_0 is equal to its long-run equilibrium value K^0.

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[3] It is known that there are some graphical methods to solve the Van der Pol type of non-linear differential equation of the second order; the method of isoclines, the Lienard Method, the E-function method, and the Kaplan method. What we are attempting here is, unlike those methods, applicable only to a certain type of non-linear difference equations. Our method has the advantage of using a familiar diagram to economists and of being applicable even to a certain type of non-linear difference equations of orders higher than two.

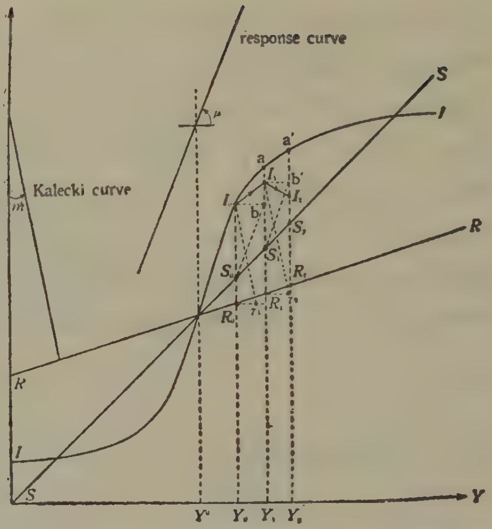

Fig. 5

In this case the investment decision in period *0* is I_0, and the excess of investment over saving is $I_0 S_0$. The level of Y_1 is, therefore, reached as before. The investment decision in period 1 is not, however, given by *a*. For the capital accumulated in the preceding period $(=I_0R_0)$ decreases the investment decision.[4] This effect may be estimated by the distance between R_0 and r_1, where the slope of I_0 r_1 is parallel to the kalecki curve. Hence, the invesłment decision is given by I_1 that is obtained by sutracting this R_0 r_1 from *a*. In this way the "Kalecki curve" may be utilized in order to determine the depressing effect of capital accumulation on investment. It may well be called the "Kalecki curve" after the first economist, Dr. kalecki, who introduced explicitly the effect into the theories of econmic fluctuations. Similarly we reach to Y_2 and obtain r_2. Then I_2 should be determined by subtracting R_0 r_1 *plus* R_0 r_2 from *a'*. This can easily be seen from equation (1). Repeating the same procedure, we can trace the course of development of investment. Fig. 6 is the graphical solution for the following system of equations:

$$I_t=f(Y_t)-K_t \qquad \cdots\cdots\cdots\cdots\cdots\cdots(1)'$$

$$S_t=0.2Y_t \qquad \cdots\cdots\cdots\cdots\cdots\cdots(2)'$$

$$R_t=0.1Y_t+7 \qquad \cdots\cdots\cdots\cdots\cdots\cdots(3)'$$

$$Y_{t+1}-Y_t=I_t-S_t \qquad \cdots\cdots\cdots\cdots\cdots\cdots(4)'$$

where $f(Y_t)=Y_t$ for $60<Y_t<80$ and $f(Y_t)=Y_t/30$ otherwise.

Fig. 7 shows the course of the change of Y_t through time. It should be observed that the kaldor model is the one which gives the

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[4] *cf.* M. Morishima, *op. cit.*, S. Ichimura, *op. cit.*

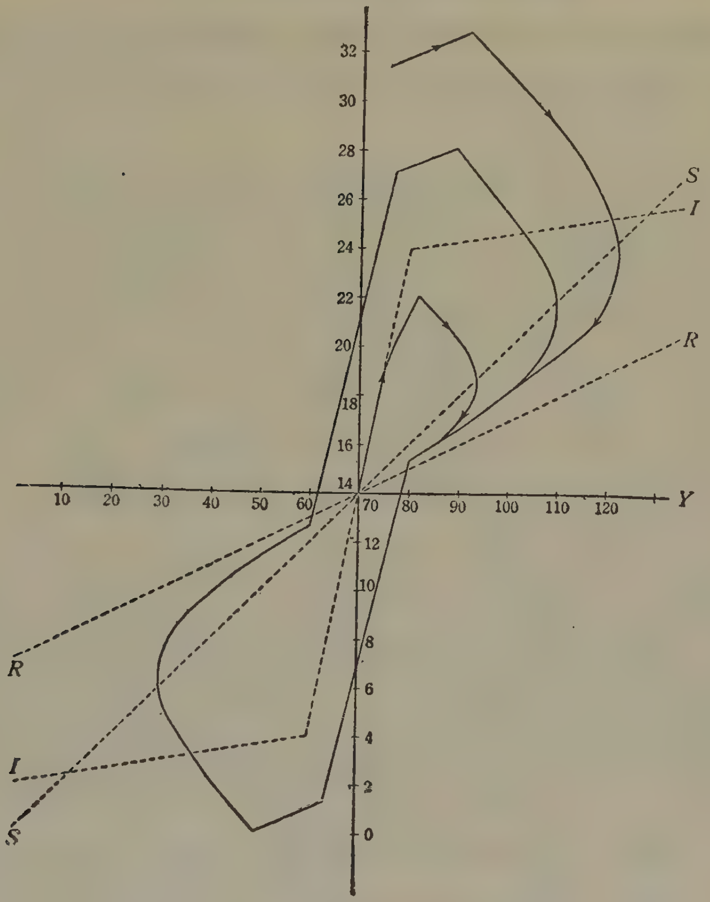
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Fig. 6

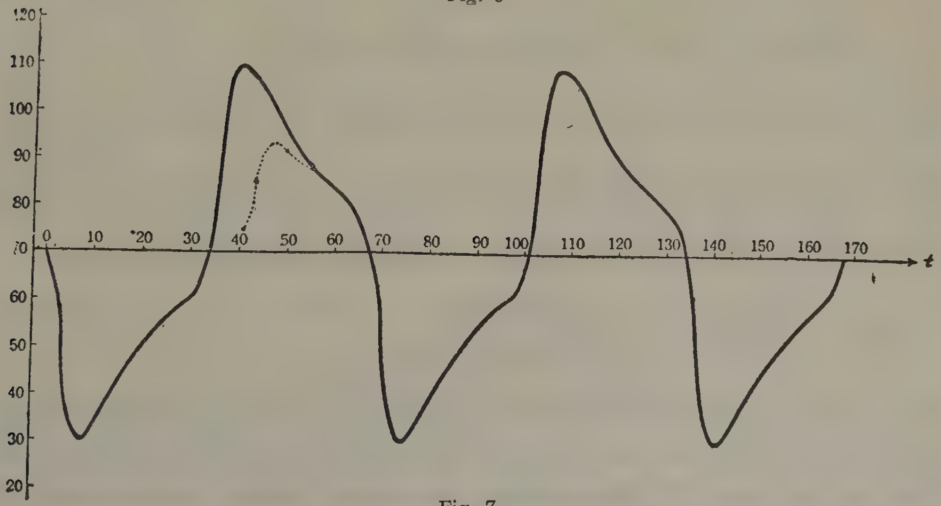
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Fig. 7

slow movement in the beginning stages of upswing and downswing. It is now clear that this model has a unique limit cycle, for example, for above-mentioned values.

4. Now we turn to the Hicks-Goodwin theory of the trade cycle. In spite of some differences in the form of the investment function, Hicks' theory and Goodwin's theory are essintially the same. Both theories adopt the *irreversible* acceleration principle for the investment functions. Furthermore, Hicks added the ceiling for the *level* of economic activity which output could not exceed, whereas Goodwin incorporated the ceiling of the levl of output into the investment function. We shall show below that these two theories can also be discussed in terms of a diagram similar to Fig. 6.

In the first place, the acceleration principle of Hicksian type will be expressed in terms of the kaldor type of investment function as follows. The acceleration principle assumes that capital in use is proportional to the level of output (Y) or at least depends on Y so that the investment schedule depends on the rate of change in Y; that is $\dot{Y}$. This principle, however, tacitly presupposes that the planned investment is realized with no delay in every phase of the cycle so that in the upswing capital in use always keeps the required relations with the level of output. Otherwise, the unrealized part of the planned investment in one period will again drive up the required rate of investment in the following period. Clearly, the strict application of the acceleration principle to every phase of the trade cycle involves some difficulty when there is some idle capital or when part of intended investment does not materialize itself. In order to avoid this difficulty, we assume that the investment schedule is equal to the required amount of capital *minus* the normally utilized amount of existing capital. Namely,

$$\begin{aligned} &I_t = F[x] \\ &x = g(Y_t) - nK_t + l_t\,. \end{aligned} \tag{6}$$

Here $g(Y_t)$ represents the required amount of capital determined by the level of economic activity; l_t the amount of investment for the long-run pruposes; K_t the existing amount of capital; n the degree of normal utilization of capital; and $F[x]=x$ for $x>l_t$, $F[x]=l_t$ for $x \leqq l_t$. For the moment we assume that l_t is constant.

In the second place, we have to take care of the problem of ceiling in Hicks' theory within the framework of our formulation. This can be done by considering the response coefficient—μ in (4)—as a function of the level of economic activity (Y) When Y exceeds a certain level, various economic difficulties due to the shortage of labor or capital will make supply stagger in response to Effective Demand. In other words, the response coefficient (μ) becomes increasing small as Y becomes greater than a certain value, say, Y_f and it will eventually be almost zero, where output stops increasing. It should be noted that if Y_f is primarily determined by the shortage of labor, Y_f is not likely to change unless population is increasing (or declining). But if Y_f is conditioned by the shortage of capital, it changes as capital available for production increases or decreases. For simplicity, however, we shall assume that the beginning of the "ceiling-belt" is determined by the shortage of labor. Assumming the same saving and replacement investment functions as before, we have the following system of equations for the reformulated Hicks-Goodwin theory:

$$(6) \quad \begin{cases} I_t = F[g(Y_t)-nK_t+l_t] \\ S_t = sY_t \\ R_t = rY_t+a \end{cases}$$

$$(7) \quad Y_{t+1}-Y_t = \mu(Y_t)(I_t-S_t)\,.$$

Then the long-run equilibrium is determined by[5]

$$(8.1) \quad g(Y_t)-nK_t+l_t = S_t$$

$$(8.2) \quad g(Y_t)-nK_t+l_t = R_t\,.$$

The behavior of this system can be seen in the following diagram similar to Fig. 6. The way of obtaining a limit cycle from a given initial situation is

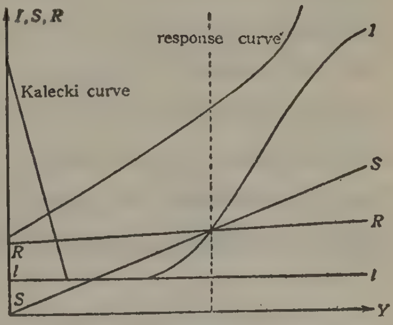

Fig. 8

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[5] We assume that $I_t=l_t$ does not give a unique solution to $I_t=S_t=R_t$.

practically the same as the method we used in Fig. 6, except that particular attention must be paid to the special shape of the investment function (6).

We shall not repeat, therefore, the explanation of the graphical method again. A possible synthesis of these two types of theories; the kaldor and the Hicks-Goodwin thsories, may be suggested here. The representative firm may be supposed to base its investment plan partly on the kaldor type of investment function and partly on the Hicks-Goodwin type of investment schedule. Then the system will have the characteristics of two theories. The solution of this system Y_t may not show a smooth mountain-type of behavior through time but have a wavy-type of behavior. The graphical method we used before can be applied to this generalized system as well. This is the advantage of our elementary method of graphical solution which even electronic computers does not have. This seems to suggest that several cycles of different periods may be due to the different patterns of behavior of various sectors in the economic system.

Part II. A New Non-Linear Theory

5. Now we sall proceed to the formulation of a non-liner macro-dynamic theory of the trade cycle from a rather different point of view. The main difference between the following theory and the above-menteioned theories is that they started from macroscopic concepts at the outset, whereas we derive the induced investment function and the short-run supply function of output from the neo-Marshallian theory. As a result we shall test the assumptions of their theories in the light of the theory of the firm. There are some startling points which we may say against their theories.[6] This theory indeed includes the kaldor theory and Hicks-Goodwin theory as special cases. Furthermore, if the

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[6] The following argumet is greatly indebted to the excellent paper by Mr. Hugh Rose; H. Rose, "Demand, Supply, and the Price-level in Macro-Economics," *Review of Economic Studies*, 1952–3, Vol. XX (1), No. 51, and the writer's unpublished paper on "The Investment Function and Theory of the Firm."

present writer is not mistaken, this general theory seems to cover some of classical trade cycle theories such as Spietoff's, Aftalion's, Robertson's and others' thoeries with respect to their central features. Finally, some remarks will be added about the relevance of this theory to the theory of economic growth so popular in Japan at the present time.

To simplify the following argument, we shall make the following assumptions.[7]

(1) Our representative entrepreneur faces the purely competitive market, whereas our representative consumer has the usual consumption function.

(2) An easy passage to temporary equilibrium is assumed in the sense that the price-level (p) is instantaneously adjusted so as to make demand equal to supply.

(3) The entrepreneur makes two decisions; the short-run output plan based upon the short-run expected price (p), and the long-run investment plan based upon the long-run exected price (P). And he expects that the price estalished in the market will continue in the short-rum, but that the long-run price is higher than the short-run price by a constant value.

(4) The ratio of the price-level of consumption goods to that of capital goods is constant, so that we have only to consider one price-level: that of output as a whole.

(5) Wages do not change significantly while there is unemployment of labor but becomes flexible from the point of full employment. The price-level we shall be talking about is the ratio of the price-level of output to the rate of money wages.

(6) Thus the price-level of capital goods is not constant in relation to the rate of money wages. But we suppose that the substitution of capital for labor or labor for capital due to the change of the relative price levels is negligible. In other words, all investment is capital-widening.

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[7] Not all of the assumptions are necessary for the following arguments. But there are some difficulties in constructing the theory without them. Thus far, the present writer at least has not succeeded in finding better hypotheses.

(7) The influence of the rate of interest upon investment, saving is sufficiently slight to be neglected. Or the rate of interest moves parallel to the level of output so that its effect upon investment and saving may be included in that of the level output or the price-level.

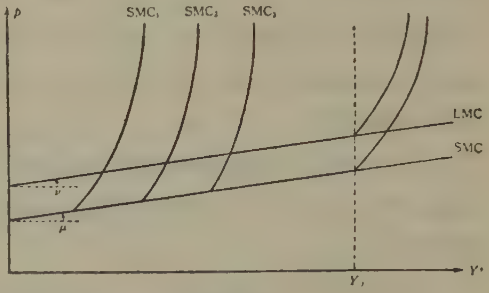

Fig. 9

Now we shall first take up the investment function. The demand for capital is determined as a part of the long-run plan. Assumming the constant returns to scale and a perfect market for loans, we know that the demand function for capital including the reservation demand is linear with respect to the long-run price P. This is likely to hold is likely to hold approximately if pure competition is prevailing in the product market; the long-run cost curve is linear; and P is large. If we ignore the effect of the change in the rate of interest on the demand for capital and assume that the period of long-term planning is equal to a unit period, we have

$$(1) \qquad K_t^* = bP_{t-1} + \beta\,, \quad \left(b = \frac{v}{\nu}\right)$$

where K_t^* represents *the required amount* of capital, and b is a positive constant, and b is the ratio of capital in use to capacity (measured in terms of output —denoted v— divided by the slop of LMC (ν). Though we have assummed a linear LMC, the LMC will start rising sharply after output reaches the point of *full employment of labor*. Moreover, the capital coefficient (v) will also become increasingly small. Remembering all this, (1) may be rewritten:

$$(1.1) \qquad K_t^* = \frac{v(Y_{t-1})}{\nu(Y_{t-1})} P_{t-1} + \delta v = b(Y_{t-1}) P_{t-1} + v(Y_{t-1})\delta\,,$$

where $\delta v = \beta$.

By assumption, $P_t = P_{t-1} + \gamma$, where γ is a positive constant. Hence,

$$(1.2) \qquad K_t^* = bp_{t-1} + b\gamma + \delta v\,.$$

So long as b remains constant, $b\gamma + \delta v$ may be regarded as autonomous in that it is independent of P_{t-1}. But once the full employment level of output is reached, it is no longer autonomous.[8]

Before deriving the investment function we have to distinguish between the existing amount of capital (K^e) and the amount of capital available for production (K^a). Even if the planned investment is realized, both are not necessarily the same thing when the construction of capital takes longer than a unit period. For before the construction is completed, the existing capital is not yet available for productive operation. Here we are concerned with K^a, not K^e.

Now using the same symbols as before, we may write the investment function as:

$$(2) \qquad \begin{cases} I_t = F[x] \\ x = bp_{t-1} + b\gamma + \delta v - nK^a_{t-1}, \end{cases}$$

where $F[x] = \frac{1}{\theta}x$ for $x > b\gamma + \delta v$; $F[x] = b\gamma + \delta v/\theta$ for $x \leq b\gamma + \delta v$. Here θ is the average period of construction of capital equipment, and it is assumed that the investment plan is uniformly spread over time. The reason why we use K^a in (2) rather than K^e is to take care of what R. Frisch called the carry-over activity of investment plan. The mere increase of K^e itself may have the depressing effect on investment. Then, we are perfectly allowed to include the term, but here we are ignoring the effect.

Next, as for the short-run out-put plan, we know that the entrepreneur bases the plan on the maximization of net revenue obtainable from operating K^a at time t. The short-run output (Y^s_t) is thus an increasing function of p_t and K^a_t. The assumption of an easy passage to temporary euilibrium permits us to ignore the time-lag between Y^s and p. Thus we have

$$(3) \qquad Y^a_t = f(P_t, K^a_t).$$

We have assumed above, in order to make b constant, that the long-run marginal cost curve is linear. This implies that the short-run marginal cost curve (SMC) for any given K^a will have the same slope as the long-run marginal cost curve up to the point of full capacity, though it will lie below because of interest and depreciation charges. Thus, before the full capacity point the short-run output schedule may be written:

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[8] Compare with Hicks' analysis; J. R. Hicks, *op. cit.*, pp. 120–123.

$$Y^s_t = \frac{1}{\mu} P_t + \epsilon . \tag{3.1}$$

It is to be noted that μ is the slope of SMC, and by assumption $\mu = \nu$. After the full employment level of output Y_f (see Fig. 9), μ and ϵ become the functions of the level of output, but μ is still equal to ν:

$$Y^s_t = \mu(Y^s_t)P_t + \epsilon(Y^s_t) . \tag{3.2}$$

From the point of full capacity the SMC will rise more steeply. (see Fig. 9)

Further to simplify the argument we make an additional assumption that the steeper portions of SMC's are all parallel. In this case the short-run output schedule after the point of full capacity has the simple form:

$$Y^s_t = \varphi(p_t) + hK^a_t , \tag{3.3}$$

where he is a positive constant and $\varphi'(p_t)$ is the reciprocal of the slope of the steeper part of SMC.

Thirdly, we have to consider the demand side again. By definition, Effective Demand (Y^D_t) is equal to consumption *plus* investment:

$$Y^D_t = C_t + I_t . \tag{4}$$

By the assumption of an easy passage to temporary equilibrium

$$Y^D_3 = Y^s_t . \tag{5}$$

Hence from now on we may write both Y^D_t and Y^s_t as Y_t, and we assume that this Y_t is the income of consumers.

Let us assume that the consumption function may be written as:

$$C_t = aY_{t-1} - eP_t + \lambda . \tag{6}$$

The term $-eP_t$ implies that when p is rising, there will be forced savings, and that when p is falling, the same effect as the Pigou effect takes place.

This completes the system of equations representing our theory of the trade cycle. In oder to compare this theory with Hick' or Goodwin's, we shall consider the following two cases: (1) the case where the normal degree of utilization of K^a: namely, n is very small or p is far greater than p so that the point of full capacity is never reached. (2) the one where the point of full capacity is reached somewhere in the middle of the course of upswing.

Case (1): the SMC curve does not reach the point full capacity, but it may well reach the point of full employment. It is convenient, however, to explain the behavior of the system in the upswing before the point of full employment and compare it with the behavior of the Hicksian system. We shall assume, for simplicity, that the period of of construction of capital goods is equal to a unit period. Then the system may be represented by the following equations:

$$
(\mathrm{I}) \quad \left|\begin{array}{ll}
Y_t = \dfrac{1}{\mu} p_t + \in & \cdots\cdots\cdots\cdots(\mathrm{I.}\ 1) \\
Y_t = C_t + I_t & \cdots\cdots\cdots\cdots(\mathrm{I.}\ 2) \\
C_t = aY_{t-1} - ep_t + \lambda & \cdots\cdots\cdots\cdots(\mathrm{I.}\ 3) \\
I_t = \dfrac{v}{\nu} p_{t-1} + \dfrac{v}{\nu}\gamma + v\delta - nK_{t-1} & \cdots\cdots\cdots\cdots(\mathrm{I.}\ 4) \\
K_t = K_{t-1} + I_t & \cdots\cdots\cdots\cdots(\mathrm{I.}\ 5)
\end{array}\right.
$$

From (I. 1) we get

$$p_t = \mu(Y_t - \in) \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(\mathrm{I.}\ 1)'.$$

This shows that p_t changes in the same direction Y_t, and that the constat price-level is only consistent with the stationary value of Y_t. If, however, μ is nearly equal zero; that is, the short-run marginal cost is almost constant, then the price is constant even though Y_t may be changing. This seems to imply that the usual assumption of the constant price-level is valid only in the phase of depression.

From (I), we can easily obtain:

$$
(\mathrm{I})' \quad \left\{\begin{array}{l}
Y_t + e\mu(Y_t - \in) = aY_{t-1} + v(Y_{t-1} - \in) - -nK_{t-1} + A \\
K_t = K_{t-1} + Y_t + e\mu(Y_t - \in) - aY_{t-1} - \lambda,
\end{array}\right.
$$

where $A = v(\gamma/\nu + \delta) + \lambda$. This system can be reduced to:

$$(1+e\mu)Y_t = \{(1+e\mu)(1-n)+a\}Y_{t-1}+v(Y_{t-1}-Y_{t-2}) \\ +a(n-1)Y_{t-2}+ne\mu\in+n\lambda\,. \qquad \cdots\cdots\cdots\cdots \text{(I. 6)}$$

This is the equation in our theory corresponding to the equation of the element case in Hick's theory. If $e=0$, $n=1$, then (I. 6) becomes:

$$Y_t = aY_{t-1}+v(Y_{t-1}-Y_{t-2})+\lambda\,.$$

This means that Hicks not only ignered the effect of the price change on consumption and the degree of utilization of capital but also did not take into account the accumulation of capital due to the autonomous investment when he formulated the investment function. *In this sense*, Kaldor's and others' criticisms on unproductivity of Hicksian autonomous investment are valid. But there is a case in which the Hicksian investment is permissible. Suppose that owing to the improvements of technology the LMC curve shifts downwards without changing the ratio of capital to output. In other words, the inventions are of the labor-saving type. Then, $v_t=v_{t-1}$ but $\delta_t-\delta_{t-1}>0$ so that (I. 6) may be written for the case of $n=1$, $e=0$:

$$Y_t = aY_{t-1}+v(Y_{t-1}-Y_{t-2})+v(\delta_{t-1}-\delta_{t-2})+\lambda\,.$$

But there is not much sense to presume that inventions are only of this type. It is interesting, however, to note that if the changes in δ are accompanied, as is very likely, by the parallel changes in $\in$, then the equilibrium values of Y and p will be changed respectively by the amounts:

$$\Delta Y = \frac{v}{1-a}\Delta\delta\,, \quad \Delta p = \mu\Big(\frac{v}{1-a}\Delta\delta-\Delta\in\Big) = \mu\Big(\frac{v}{1-a}-1\Big)\Delta\delta\,.$$

Once the full employment level of output is reached, μ, $\in$, $\frac{1}{v}$, ν and δ will all become increasingly larger as Y increases. In this phase, (I)′ may be written as:

$$\text{(I)}'' \quad \begin{cases} M(Y_t)Y_t = aY_{t-1}+\lambda+F[g(Y_{t-1})-nK_{t-1}] \\ K_t = K_{t-1}+Y_t+e\mu(Y_t)\{Y_t-\in(Y_t)\}-aY_{t-1}-\lambda\,, \end{cases}$$

where

$$M(Y_t) = \left\{1 + \frac{e\mu(Y_t)\{Y_t - \epsilon(Y_t)\}}{Y_t}\right\},$$

$$g(Y_{t-1}) = v(Y_{t-1})\{Y_{t-1} - \epsilon(Y_{t-1})\} + A(Y_{t-1}).$$

It may safely be assumed that $M'(Y_t) \geqq 0$, $g''(Y_{t-1}) \leqq 0$.

The method of graphical solution we have used before is again applicable to this equation. The interested reader may work this out. It would be unnecessary to point out that (I)″ holds for any phase of the trade cycle. If care is taken of the shapes of the functions involved.

Case (2): It is too optimistic to suppose that investment is planned and fulfilled so much ahead that the point of full capacity is never reached. In fact, it is more natural to suppose that investment starts when the full capacity point is almost reached, and that the rapidly increasing demand must be met by the insufficient supply due to the shortage of capital at some phase of the upswing. In such a case we have the following system;

$$\text{(III)} \quad \left\{ \begin{array}{ll} Y_t = \varphi(p_t) + hK_t & \cdots\cdots\cdots\cdots \text{(II. 1)} \\ Y_t = C_t + I_t & \cdots\cdots\cdots\cdots \text{(II. 2)} \\ C_t = aY_{t-1} - ep_t + \lambda & \cdots\cdots\cdots\cdots \text{(II. 3)} \\ I_t = F\left[\frac{v}{\nu}p_{t-1} + \frac{v}{\nu}\gamma + v\delta - nK_{t-1}\right] & \cdots\cdots\cdots\cdots \text{(II. 4)} \\ K_t = K_{t-1} + I_t & \cdots\cdots\cdots\cdots \text{(II. 5)} \end{array} \right.$$

For simplicity let us assume that Y_t does not reach the full employment level, so that v, ν and δ may be regarded as constants. For the following argument it is more convenient to express (II. 1) as

$$Y_t = \frac{1}{\mu'(Y_t)}p_t + hK_t \qquad \cdots\cdots\cdots\cdots \text{(II. 1)}',$$

where $\mu'(Y_t)$ is the slope of the SMC. From this we get

$$p_t = \mu'(Y_t)(Y_t - hK_t) \qquad \cdots\cdots\cdots\cdots \text{(II. 1)}'',$$

which shows that the accumulation of capital has the depressing effect on the price-level.

Then, if $K_t^* > nK_{t-1}$, and $\mu'(Y_t)$ is constant. (II) may be rewritten as;

$$(\mathrm{II})' \quad \begin{cases} (1+e\mu')Y_t = \left(a+\frac{v}{\nu}\mu'\right)Y_{t-1} - \left(\frac{v}{\nu}h\mu'+n\right)K_{t-1} + e\mu'hK_t + A' \\ (1+e\mu'h)K_t - K_{t-1} = Y_t - aY_{t-1} - \lambda\,, \end{cases}$$

where $A' = \frac{v}{\nu}\gamma + v\delta + \lambda$.

It can be seen that if $n=1$, $e=0$, (II)$'$ reduces to:

$$Y_t - \left\{a + \frac{v}{\nu}\,\mu'(1-h)\right\}Y_{t-1} + \frac{v}{\nu}\mu'(+ah)\,Y_{t-2} = \lambda(1-vh) \cdots (\mathrm{II.}\ 6)$$

If there is some autonomous investment, it may be added to the right-hand term. This is the equation in Case (2) corresponding to that of the elementary case in Hicks' Fundamental Equation. This will be identical with Hicks' only if $\nu = \mu'$ and $h=0$. This is what Hicks tacitly assumed.

With no such special assumptions (II. 6) may be solved. Then the behavior of our system in Case (2) will be shown by our graphical method. We must, however, waive to show the solution to the next chance and satisfy ourselves by indicating briefly the way of formulating a non-linear macro-dynamic theory on the basis of the neo-Classical theory of the firm.

(September, 1953)